Introduction to Computer Graphics
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04/02 Introduction & projective rendering
11/02 Procedural modeling, Interactive modeling with parametric surfaces
25/02 Introduction to OpenGL + lab: first steps & modeling
04/03 Implicit surfaces 1 + lecture/lab: transformations & hierarchies
11/03 Implicit surfaces 2 + Lights & materials in OpenGL
18/03 Textures, aliasing + Lab: Lights & materials in OpenGL
25/03 Textures in OpenGL: lecture + lab
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29/04 Animating complex objects + Realistic rendering
06/05 Talks: results of cases studies
Implicit Surfaces
Modeling complex shapes?

- Smooth surfaces
- Branching shapes
- Local details

Three possible strategies
1. Lots of simple skeletons
2. Fewer, complex skeletons
3. Discrete scalar field
Implicit Surfaces
Modeling complex shapes?

1. Lots of simple skeletons, additive field

- Cost of field queries function of the number of skeletons
  Can be improved using local fields plus a query grid…
- Lack of smoothness? Unwanted blending?
Implicit Surfaces
Modeling complex shapes?

2. Fewer, complex skeletons

• Intuitive
  – Skeleton graph of the shape’s topology
  – Analogy with medial axis

• Major problems!
  – Bulges
  – Unwanted blending
The bulges problem

Field values based on distance

- Distance to the closest point on $S_i$
- The shape changes if $S_i$ is divided
- 3D bulge at all joints!
Avoid Unwanted Bulges?

**Convolution surfaces** [Bloomenthal Shoemake 91]

- Integral along $S_i$ on point contributions

$$F(S, p) = \int_{s \in S} f(s, p) ds$$

= convolution of the skeleton with a “kernel” $f$
Convolution Surfaces

- [Bloomenthal Shoemake 1991]
  - Blinn’s exponential field
  - Discrete, approximate computation of the integral

- [Sherstyuk 1998-1999]
  - Some analytical solutions
  - Skeletons
    - segments
    - triangles
    - arcs of circles
Example of analytical solution  [Cani Hornus 2001]

Along a line-segment

\[ f(P) = \int_{r_1}^{r_2} \frac{1}{r^3} \, dr \]

\[ F(P) = \frac{\sin(\alpha_1) - \sin(\alpha_2)}{d^2(P,H)} \]
Convolution surfaces
Non-constant radius?

- $R(u)$ computed by interpolation
- Modified convolution kernel (iso=0.5)

$$F(P) = \frac{\sin(\alpha_1) - \sin(\alpha_2)}{D^2(P, H)}$$

where

$$D(P, H) = \sqrt{2} \frac{d(P, H)}{R(H)}$$
Surfaces of non-constant radius
Exact solution [Hornus Cani Angelidis 2002]

- interpolation linéaire de $r$

\[ f(s, p) = \frac{r^2}{d^2(p,s)}c \]

- convolution de

\[ F(S,p) = \frac{((dC-D^2/d)A+(r_0 - r_1 )DB+(a_0 - a_1 )C)}{(a_0 - a_1 )^2}c \]

\[ A=\arctan( a_1 /d)+\arctan(-a_0/d) \]
\[ B=\log((a_1^2 + d^2)/(a_0^2 +d^2)) \]
\[ C=(r_0 - r_1 )^2 \]
\[ D=r_0 a_1 + r_1 a_0 \]
Surfaces of non-constant radius

Not exact on triangles [Angelidis Cani 2002]

- Sherstuyk’ field [She98]
  \[
  F(S, p) = \int_{s \in S} \frac{1}{(1 + s^2 d^2 (p, s))^2} dS
  \]

- Practical solution to varying radius
  \[
  \frac{r_0 + r_1 + r_2}{3} F(S, p)
  \]
The unwanded Blending problem

• Primitives blend according to their distance!
Solution to Unwanded Blending

Blending graph expressing the topology

- [Guy Wyvill 1995]
  Field at P: Find skeleton with main influence
  Add its immediate neighbors: discontinuous!!

- [Cani Hornus 2001]
  Blend until the contribution is small enough
  Only works in specific cases!
Solution to Unwanded Blending

Blending graph expressing the topology

• [Angelidis Cani 2002]
  – decay functions (force the field contributions to vanish)
Example of use [Angelidis Cani 2002]

Subdivision curves & surfaces as skeletons

Implicit surfaces with levels of detail!
Implicit Surfaces
Modeling complex shapes?

1. Lots of simple skeletons
2. Fewer, complex skeletons

3. Discrete scalar field
   Discrete field, stored in a grid
   Smooth interpolation
   → Constant time field queries
Discrete scalar field: data structures

**Unbounded grid?**

- H-table storage of non-empty cells
  - Created or deleted when and where needed
  - Field values clamped to [Max, 0], where iso = max/2

**3 structures**

- non-empty cells, surface cells, surface vertices (on edges)
Sculpting tools

Analytical or discrete tools defined by

1. A continuous field function = Tool’s contribution
2. An action = The blending operation to apply

Possible actions

- Add material (+)
- Remove material (-)
- Paint
- Smooth (low band filter)
Visualization

• Real-time, incremental marching cubes
  – Re-compute triangle data only where needed

• Environment textures
  – good perception of the shape
“Push” the material with a rigid tool?
- Geometric deformation mimicking physics

\[ v_{\text{tool}} \]

iso
distance to the centre

But clay also deforms!
Addition of local deformations

- Push material with a rigid tool [Ferley Cani Gascuel, 2001]
  - User controlled bulge: no exact volume preservation
“Feel” the material: Force Feedback

Two forces are available

- Viscous friction from the tool’s speed and field value
- Contact force along the field’s gradient at the tool center

Phantom desktop
(1000 Hz)
Force feedback: Best combination of forces?

Non-applied tool

- More contact force
  - The user feels the surface
  - He can place the tool from it

Applied tool

- More viscous friction
  - The tool can penetrate inside
  - The user feels the density of clay

→ «Expressive» haptic feedback!

Sculpted in 1 hour
Multi-resolution?

- Both coarse and fine features at interactive rates?
- Internal representation still transparent to the user

**Multi-grid, two options**

1. Store average + delta contributions at finer levels
   - Elegant but extra cost for coherency
2. Store extra field samples at finer levels
   - Levels can be updated independently

→ *option 2*
Multi-Resolution

- Multi-grid of un-bounded resolution (H-table)
- The tools’ sharp features guide local resolution
- Progressive action of tools: update from coarse to fine levels
- LODs for surface display

[Tool image]

[Ferley Cani Gascuel, GMOD 2002]
Multi-Resolution

Sculpted in 1,5 hour
Modeling complex shapes

Best implicit representation?

Constructive representation (tree)
- Lots of very simple primitives?
  → objects breaking into pieces
- Fewer, complex skeletons?
  → skeleton-based animation

Discrete field, with smooth interpolation?
- Constant time field queries
- No limitation of complexity!
  → cannot be deformed easily
Advanced bibliography

Animation of Deformable Models Using Implicit Surfaces

[Cani Desbrun 1997] (SIGGRAPH 93/95]

- Precise contact modeling
- Constant volume
- Controlled blending
Advanced bibliography

Modeling with Implicit Surfaces that Interpolate

[Turk, O’Brien, SIGGRAPH 2002]

- Introduction of Variational implicit surfaces
  - Defined by solving a linear system of position constraints
  - Now widely used for reconstructing/re-sampling point sets
Advanced bibliography
2D potential fields for advanced implicit modeling

- Controllable blending defined by a free-form curve $G(f_1, f_2)$
- Unifies CSG operators & smooth blending