

# Computer Graphics: Modeling

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# Today's planning

1. Problem solving
  - 1 hour debate, 4x10 minutes presentation
  - Pros and cons of **surface** representations
  - **Volume** representations

# Teams (tentative)

- **Team 1:**

Reynald Arnerin, Azim Azma, Laurent Belcour, Robert Conceivo Da Silva, (Adrien Gomez Espana Pecker)

- **Team 2:**

Nicolas Esteves, Sylvain Guglielmi, (Juan Alberto Lahera Perez,) ~~Mohamed Riadh Trad~~, Xue Bing

- **Team 3:**

Pierre Arnaud, Varun Raj Kompella, Anja Marx, Jorge Pena, Thibault Serot

- **Team 4:**

Antoine Bautin, (Adrien Brilhault,) Noura Faraj, Alfonso Garcia, Stefano Sclaverano

# Today's planning

- ~~Problem solving~~
  - ~~1 hour debate, 4x10 minutes presentation~~
- Pros and cons of **surface** representations
- **Volume** representations

# Pros and cons of surface representations

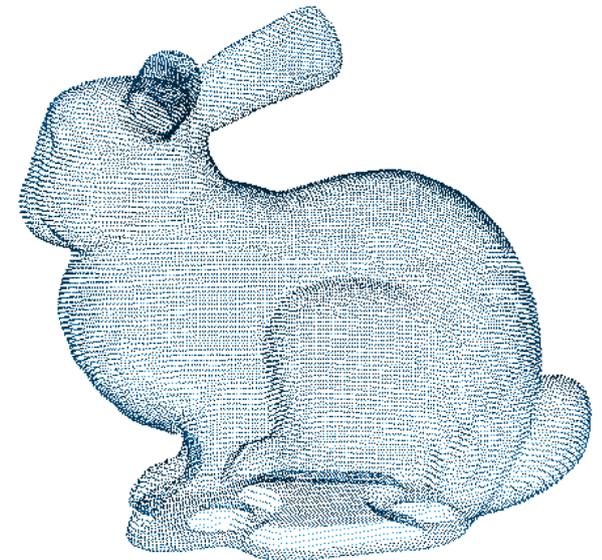
1. Point sets
2. Meshes
3. Parametric surfaces
4. Subdivision surfaces
5. Implicit surfaces

# Point sets

- Result of scanner acquisition
- Also image-based modeling
- Main **advantages**:
  - “Natural” representation
  - Simple and cheap to display
- Main **drawbacks**:
  - No connectivity info:  
underlying shape = ?
  - Tedious to edit



NextEngine scanner:  
available here!



# Too simple ?

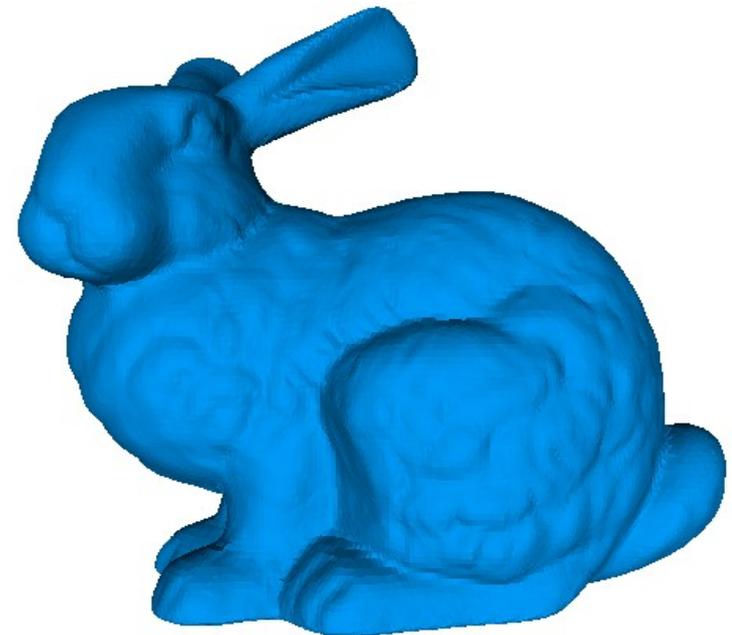
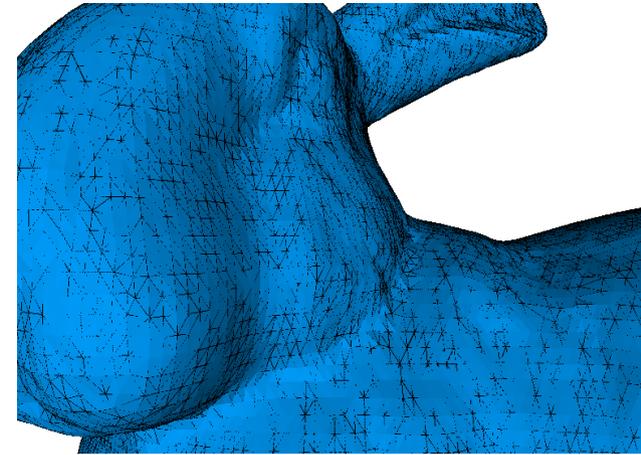
- If nb of points too low: holes
- However:
  - Currently scanned models have up to **several millions points**
  - Mesh reconstruction is then **time-consuming**
  - **Memory** to store the mesh also a problem (number of faces  $\sim 2 \times$  number of points)
  - Each face projects onto only **one or two pixels !**
- That is why surface representation by a point set is more and more used and studied

# Pros and cons of surface representations

1. ~~Point sets~~
2. Meshes
3. Parametric surfaces
4. Subdivision surfaces
5. Implicit surfaces

# Meshes

- Main **advantage**: easy display
- Main **drawback**: tedious to edit
- Represent continuous piecewise linear surfaces
- Encode
  - (Approximate) **geometry**
    - OK for planar shapes (CAD)
    - Bad for curved shapes
  - **Topology**



# Pros and cons of surface representations

1. ~~Point sets~~
2. ~~Meshes~~
3. Parametric surfaces
4. Subdivision surfaces
5. Implicit surfaces

# Why do we need Smooth Surfaces ?

## *Meshes*

- Explicit enumeration of faces
- Many required to be smooth!
- Smooth deformation???

## *Smooth surfaces*

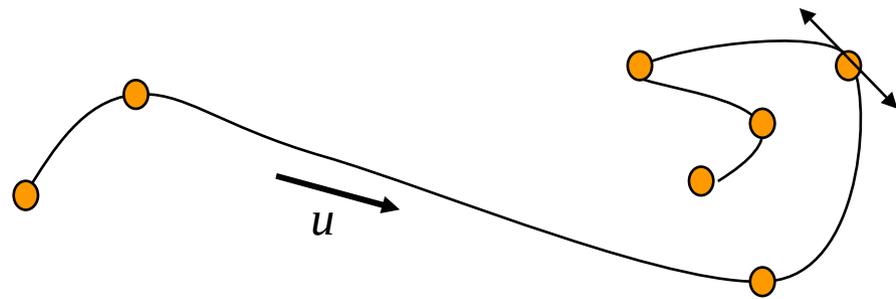
- Compact representation
- Will remain smooth
  - After zooming
  - After any deformation!



# Parametric curves and surfaces

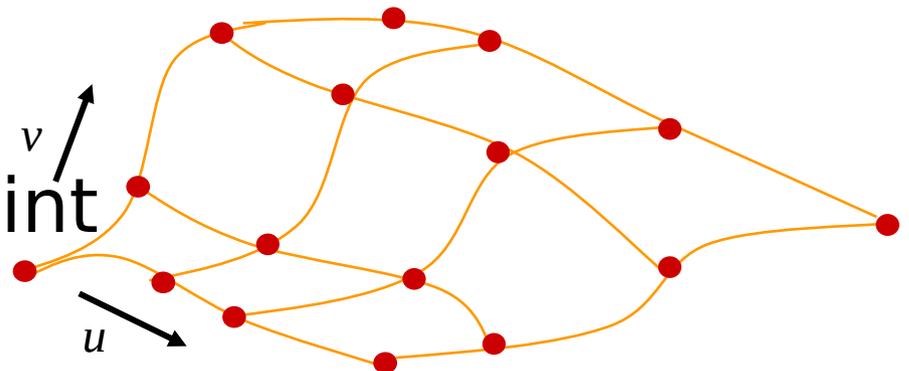
Defined by a parametric equation

- Curve:  $C(u)$
- Surface:  $S(u, v)$



Advantages

- Easy to compute point
- Easy to discretize
- Parametrization



# Pros and cons of surface representations

1. ~~Point sets~~
2. ~~Meshes~~
3. ~~Parametric surfaces~~
4. Subdivision surfaces
5. Implicit surfaces

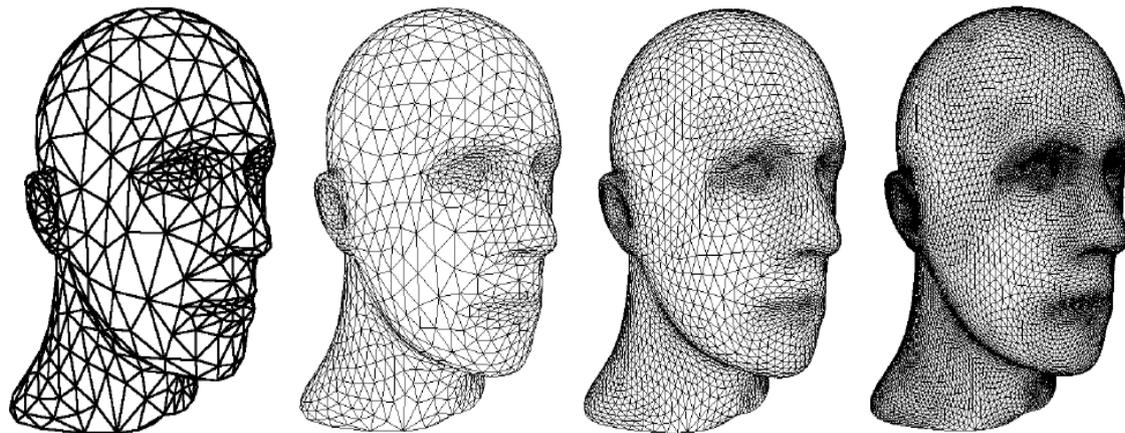
# Subdivision Surfaces

## Benefits

- Arbitrary topology & geometry (branching)
- Approximation at several levels of detail (LODs)

Drawback: No parameterization, some unexpected results

**Loop**



Extension to multi-resolution surfaces : Based on

# Pros and cons of surface representations

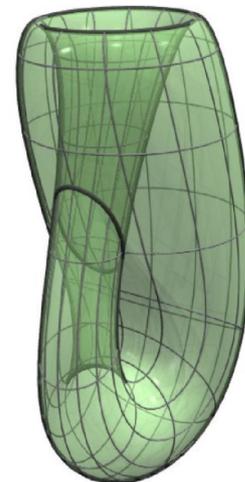
1. ~~Point sets~~
2. ~~Meshes~~
3. ~~Parametric surfaces~~
4. ~~Subdivision surfaces~~
5. **Implicit surfaces**

# Drawbacks of Boundary Representations

- **Complex shapes with splines ?**
  - Branches ?
  - Arbitrary topological genus ?

*Partly solved by subdivision surfaces*
- **Surrounding a volume?**
  - Avoid Klein bottles!
  - Prevent self-intersections

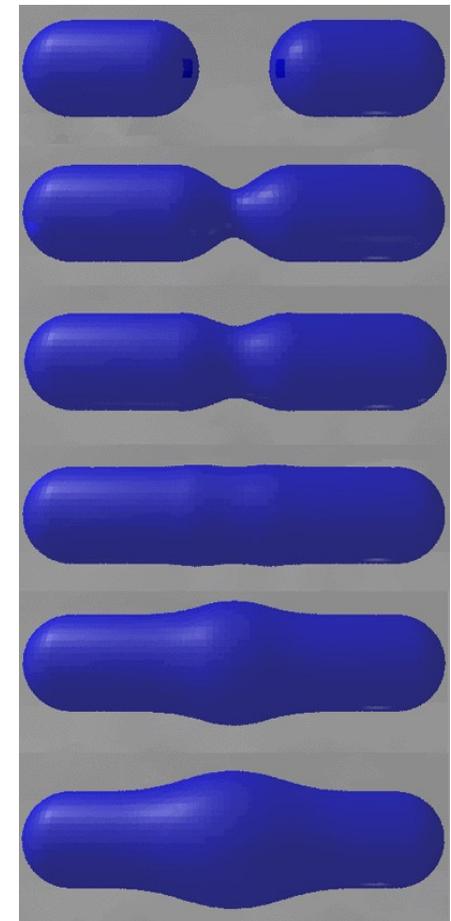
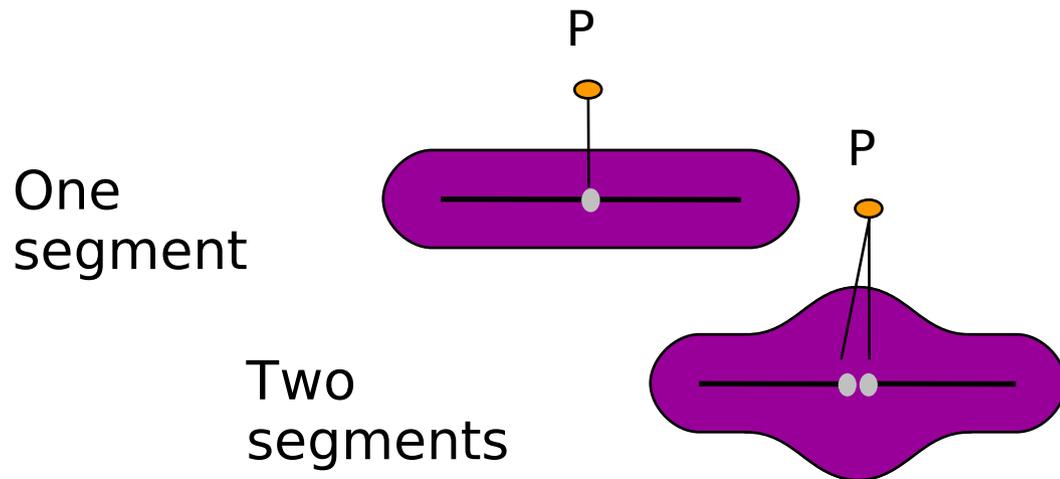
*Make them impossible?*



# Unwanted Bulges?

## *Distance surfaces*

- Distance to the closest point on  $S_i$
- The shape changes if  $S_i$  is divided
- 3D bulge at all joints!

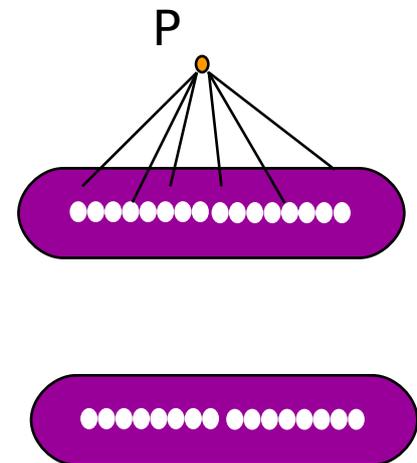


# Avoid Unwanted Bulges?

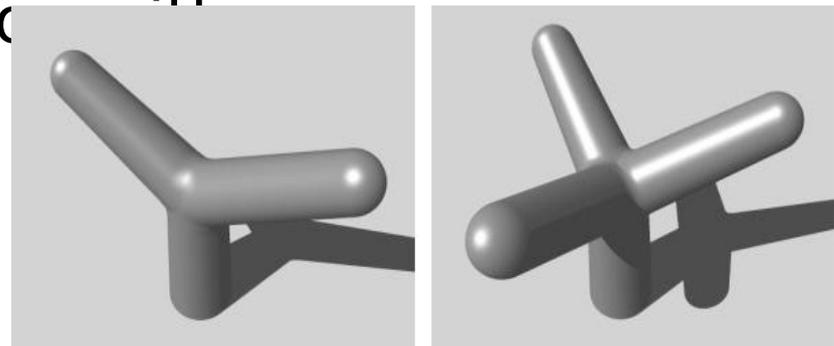
## Convolution surfaces [Bloomenthal

Shoemake 91]

- Integral along  $S_i$  on point contributions
- $$P(S, p) = \int_{s \in S} f(s, p) ds$$

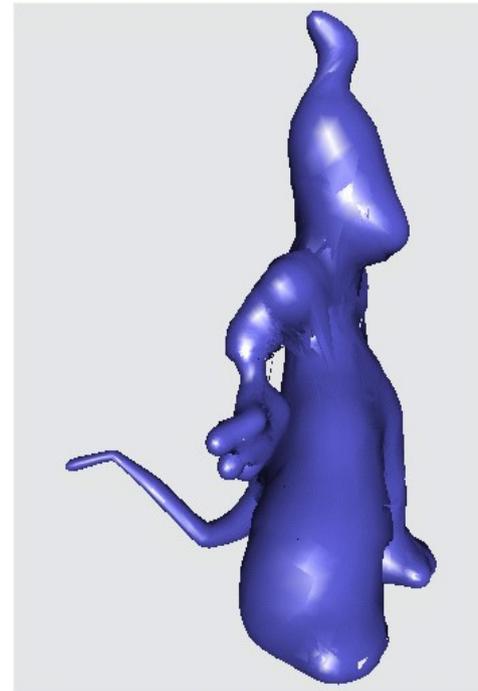
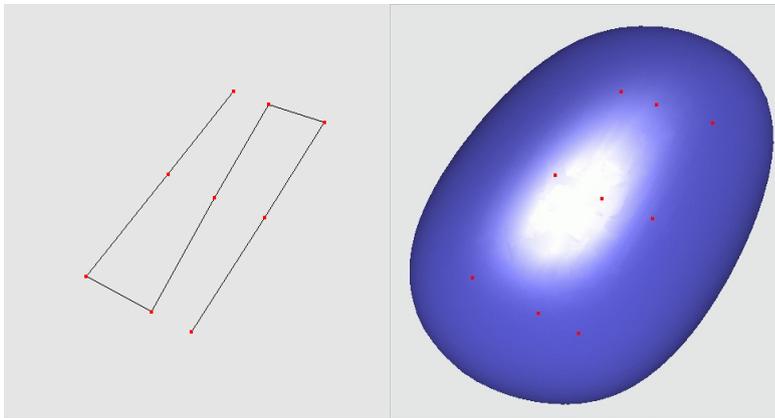


= convolution of the skeleton with kernel



# Unwanded Blending problem

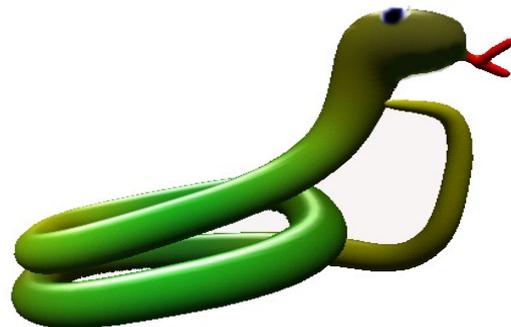
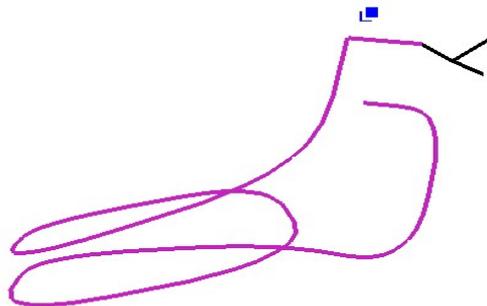
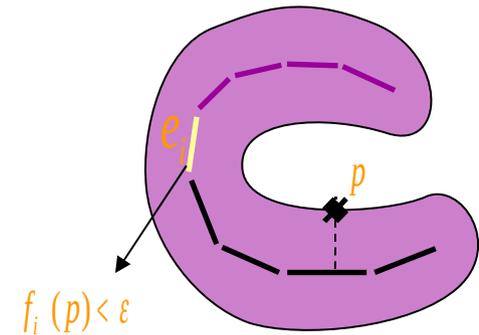
- Primitives blend according to their distance!



# Solutions to Unwanded Blending

Idea: “blending graph” expressing the shape’s topology

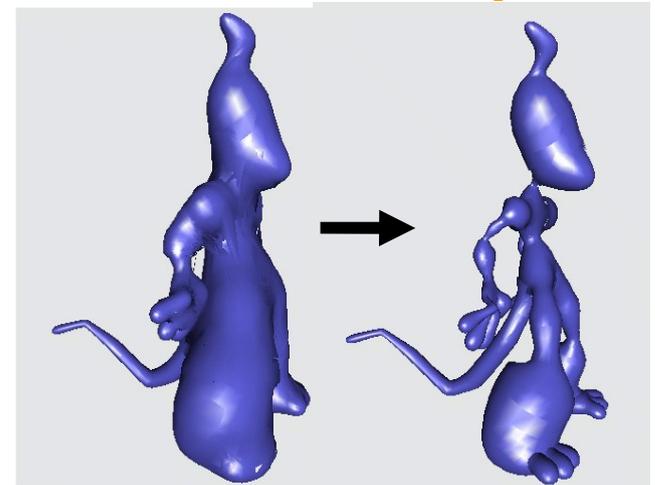
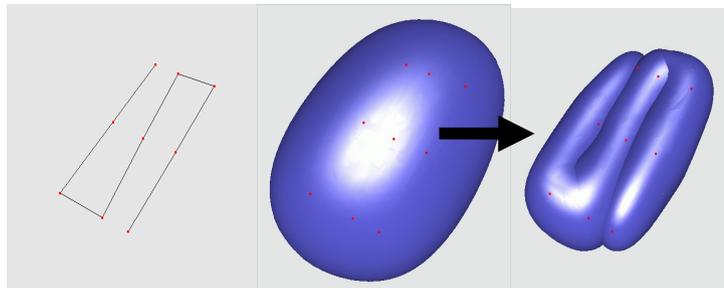
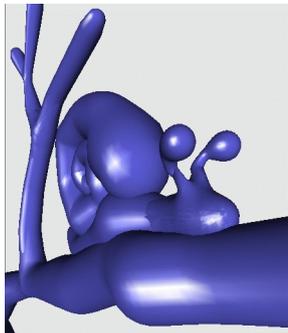
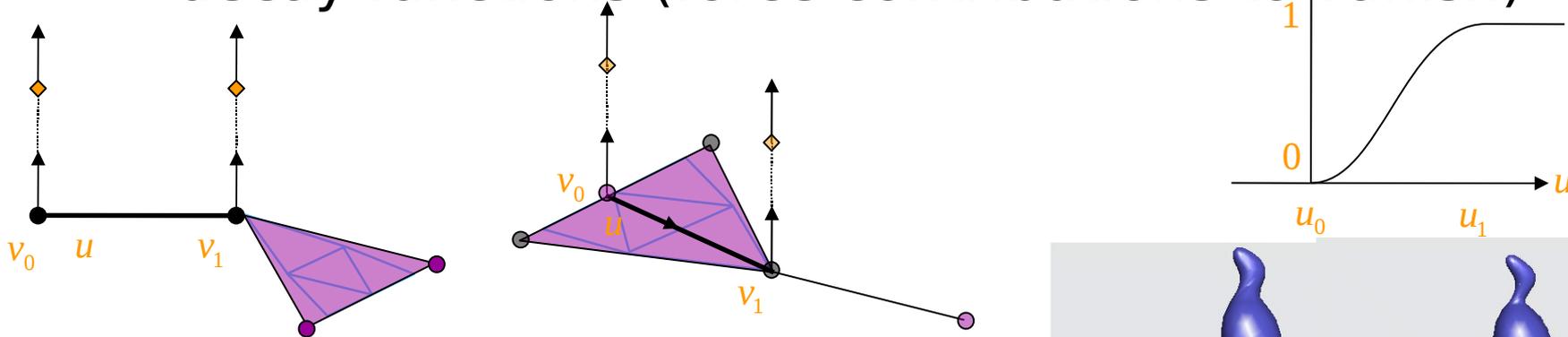
- [Guy Wyvill 1995]
  - Find the main primitive
  - Add its immediate neighbours
- [Cani Hornus 2001]
  - blend until the contribution is small enough



# Solutions to Unwanded Blending

Idea: “blending graph” expressing the shape’s topology

- *[Angelidis Cani 2002]*
  - decay functions (force contributions to  $\alpha$  vanish)

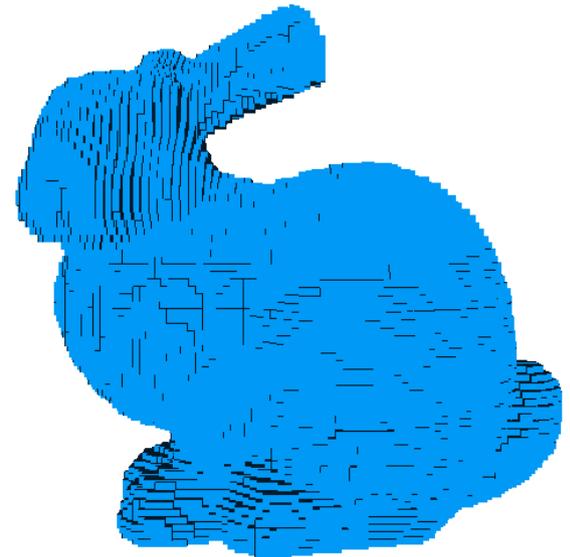
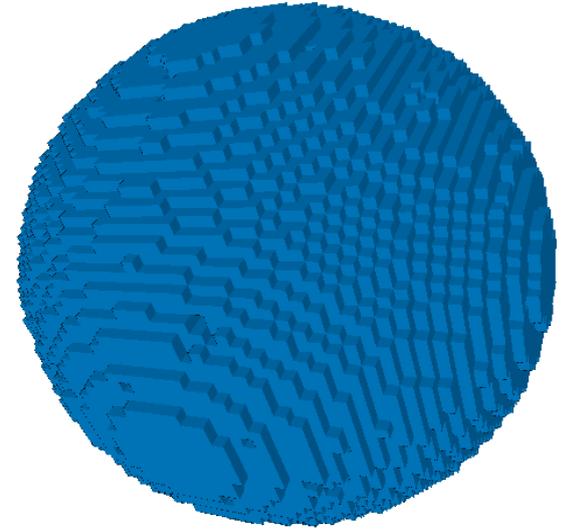


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- ~~Pros and cons of surface~~ representations
- **Volume** representations

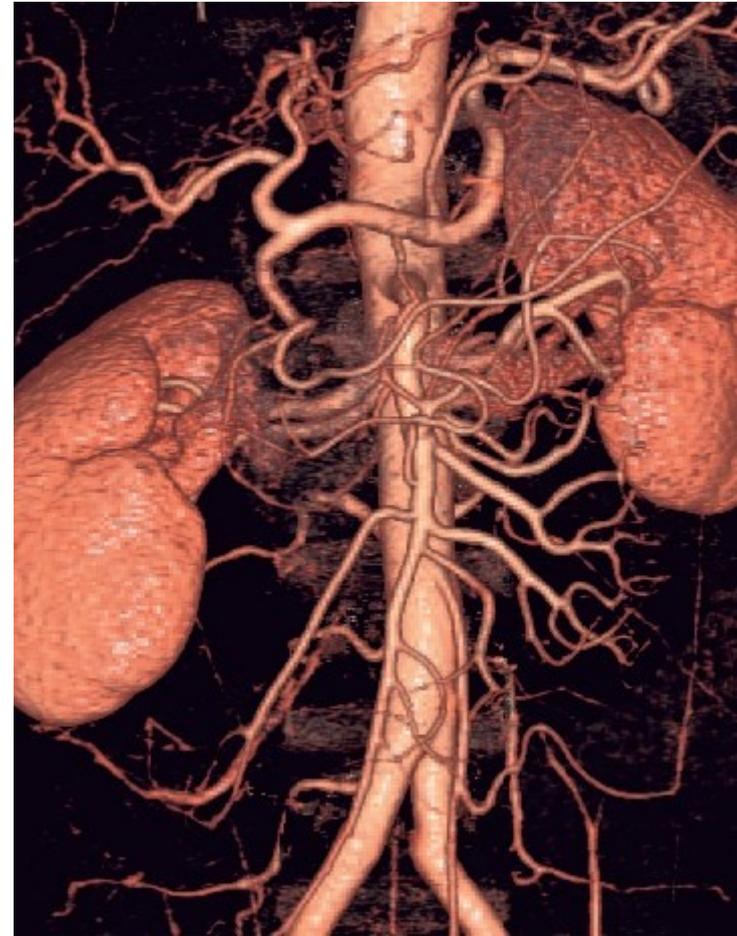
# Voxels

- **Volumetric** representation
- (Regularly) discretize the 3D space and only keep elements **inside** the object
- 2D : pixel = PICTURE ELEMENT
- 3D : **voxel** = VOLUME ELEMENT
- And also: surfel (surface), texel (texture), ...



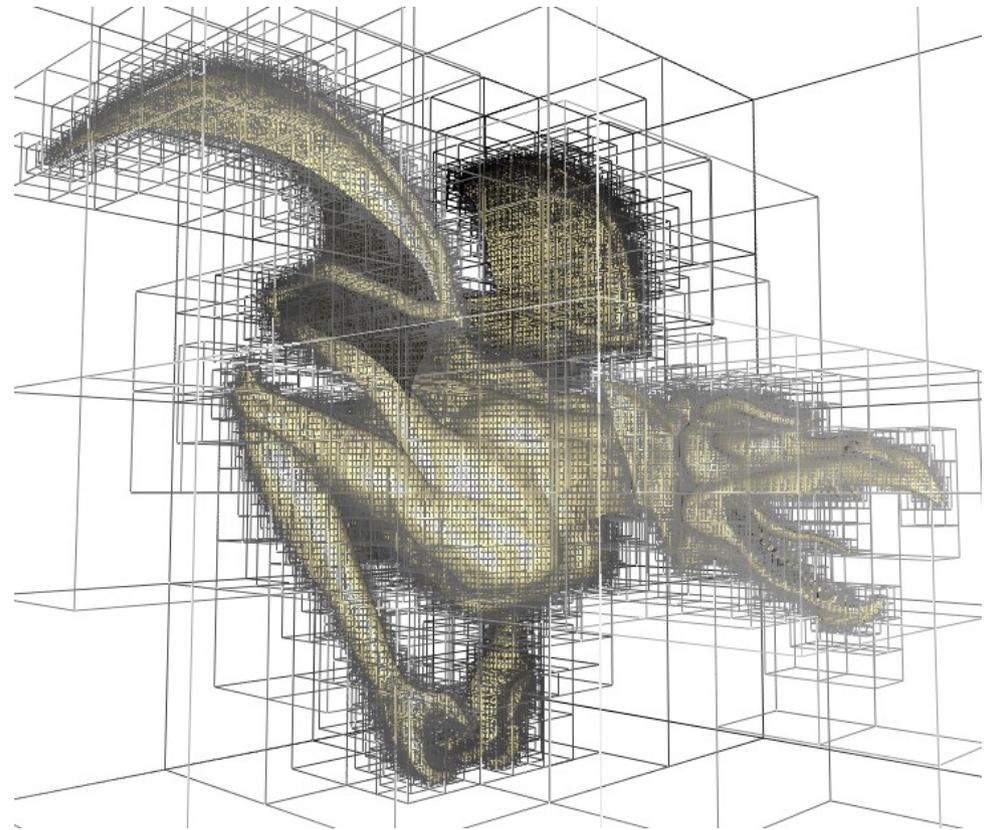
# Voxel set acquisition

- Using a function sampled on a grid
  - Numerical simulation
- Tomographic reconstruction (CT scan)
  - Medical area
- Depending on the acquisition/application, voxels contain **scalar values** (function, density, color, ...)



# Octree

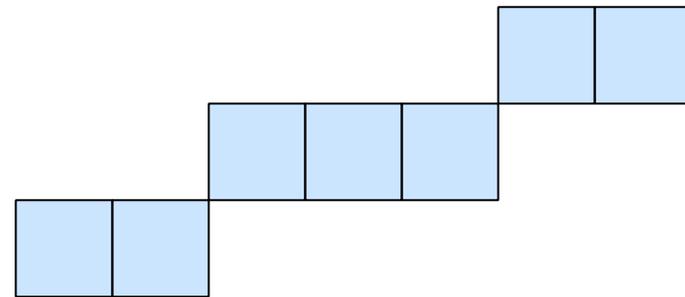
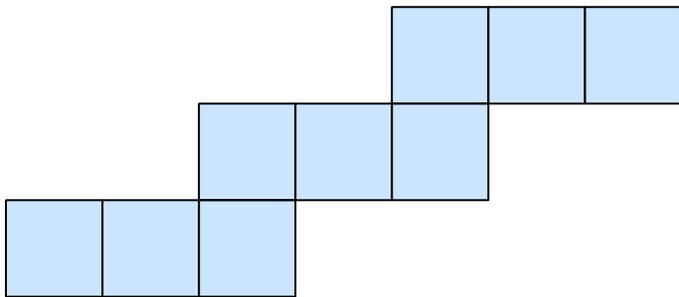
- Voxel hierarchy
- Saves memory
- Interesting for:
  - Spatial queries
  - Collision detection
  - Hidden surface removal (“view frustum culling”)



Courtesy S. Lefebvre

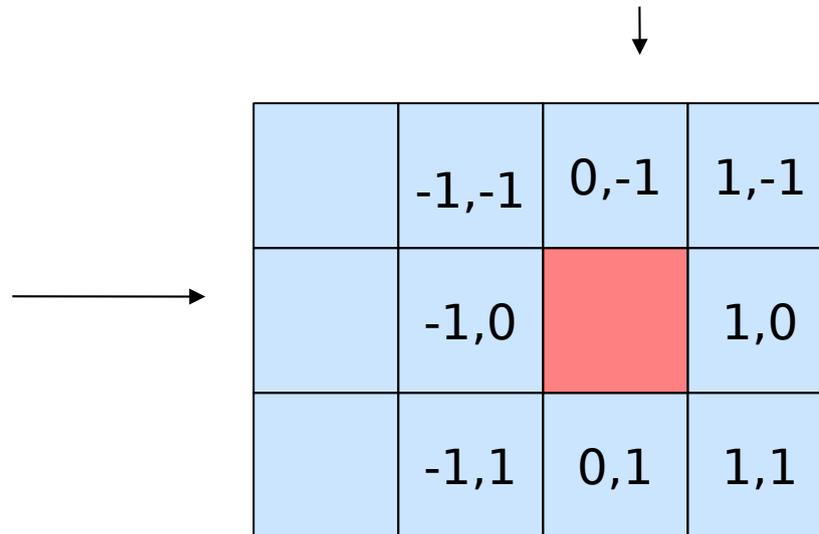
# An introduction to discrete geometry

- Theoretical/Mathematical study of **regular** 2D/3D (simple) objects
  - Sampled on a grid
  - Object = point, line, plane
- How to **define** what is a line of voxels ?
- Adapted algorithms



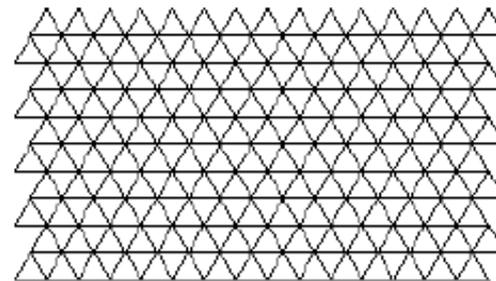
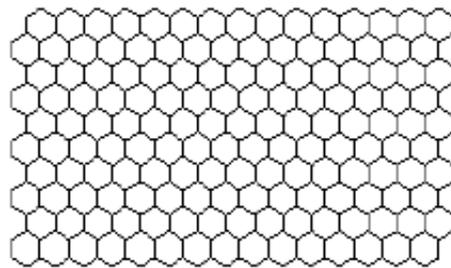
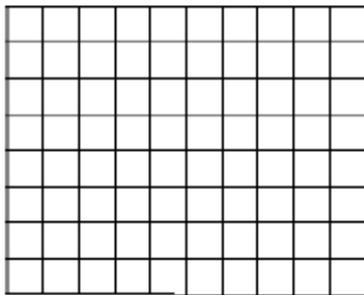
# Why a regular grid

- Simple topology
- Easy address to a cell: coordinates
- Easy access from a cell to its neighbors
- Physical reality (sensors)



# Cell

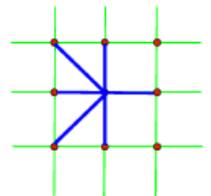
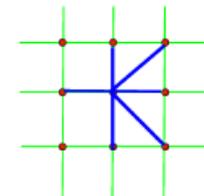
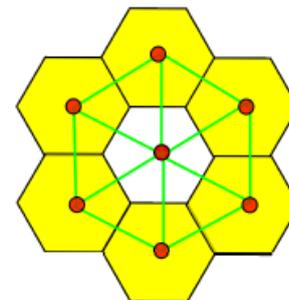
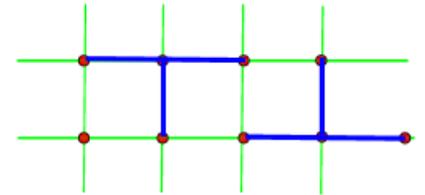
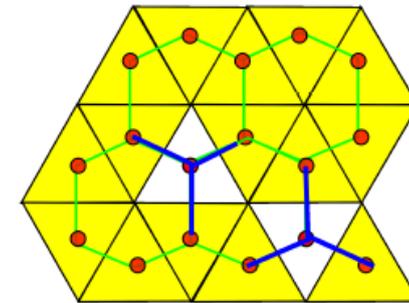
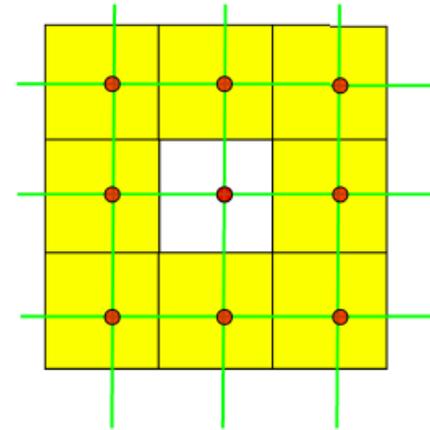
- Usually a convex polygon/polyhedron
- Regular
- The 3 principal cases: square/cube, hexagon/hexahedron, triangle/tetrahedron



Courtesy D. Coeurjolly & I. Sivignon

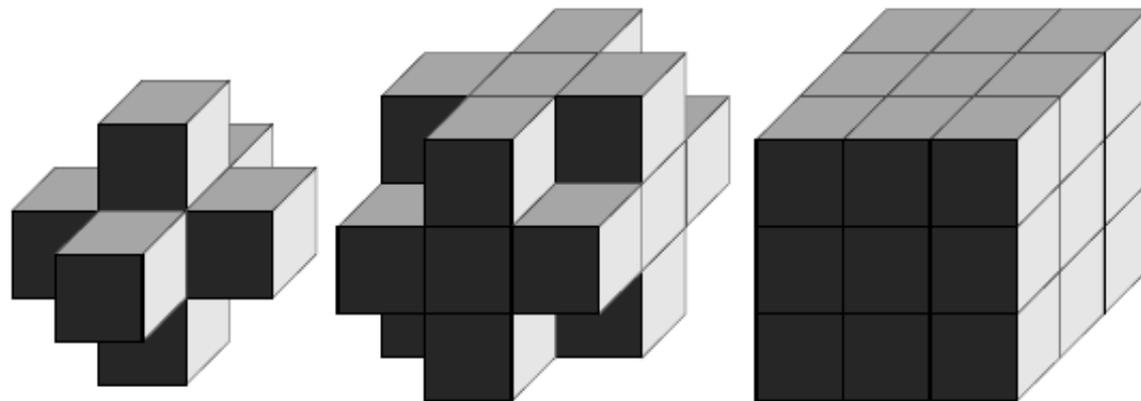
# Advantage of squares/cubes

- Square:
  - 4 neighbors
  - **1** configuration
- Triangle:
  - 3 neighbors
  - 2 configurations
- Hexagon:
  - 6 neighbors
  - 2 configurations



# Adjacency on a voxel grid

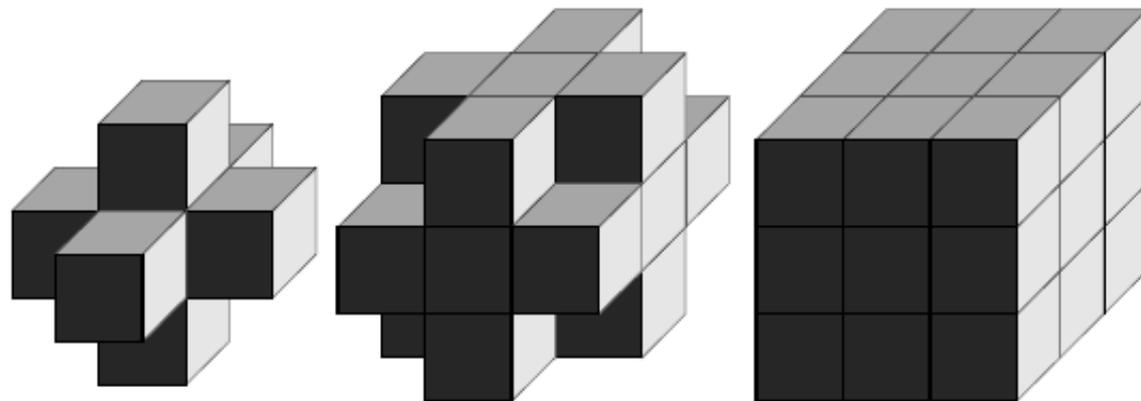
- (Combinatorial) Def.:
  - 6-neighbors = voxels that share a **face**
  - 18-neighbors = voxels that share a **edge**
  - 26-neighbors = voxels that share a **vertex**



Courtesy D. Coeurjolly & I. Sivignon

# Adjacency on a voxel grid

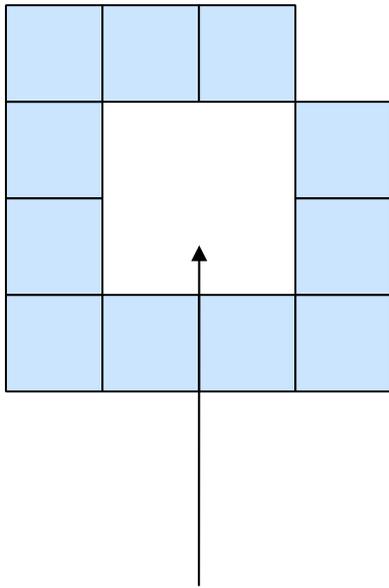
- (Topological) Def.:
  - **2-neighbors** = voxels that share a **face**
  - **1-neighbors** = voxels that share a **edge**
  - **0-neighbors** = voxels that share a **vertex**



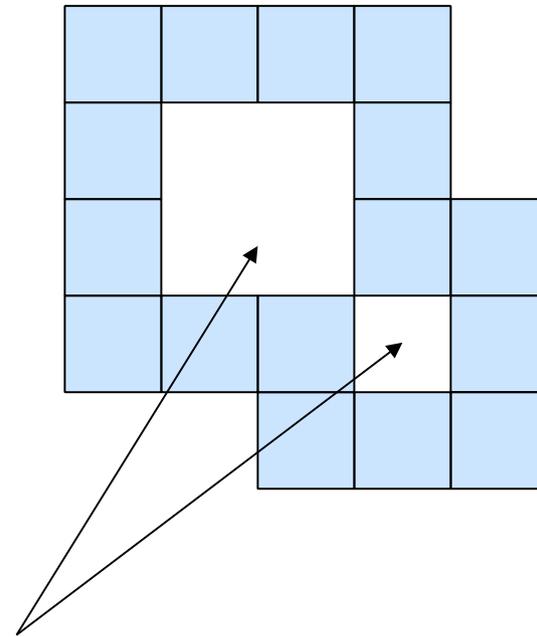
Courtesy D. Coeurjolly & I. Sivignon

# Discrete object boundary

- Problem with discrete objects: their boundary is not obvious



Inside or outside ?



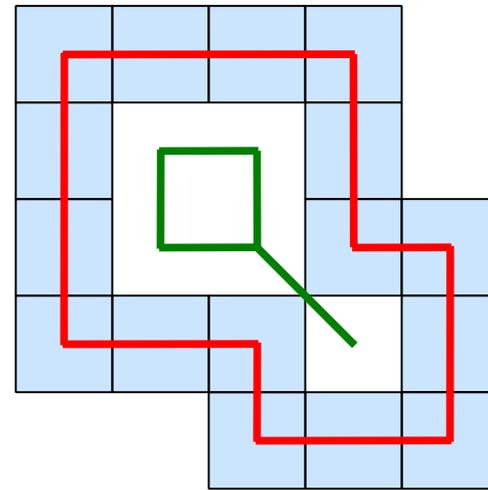
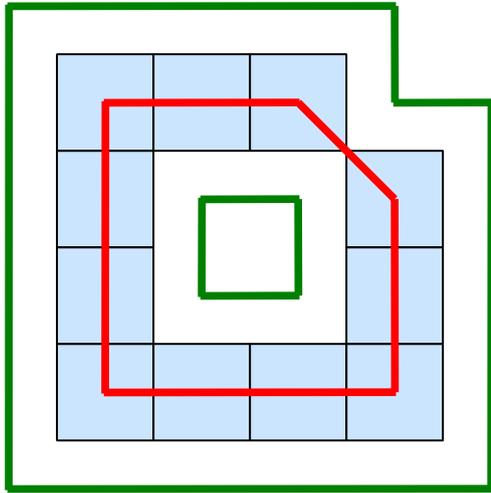
One or two components ?

# Problem

- **Jordan's theorem:** every smooth  $(n-1)$ -manifold in  $\mathbb{R}^n$  disjoints space into two connected domains (the **inside** and the **outside**); it is the common **boundary** of these domains
- **Corollary:** impossible to find a path from inside to outside
- Need to define the right adjacency !

# Adjacency couple

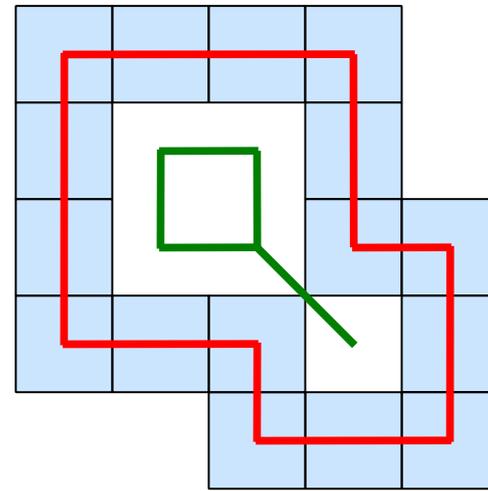
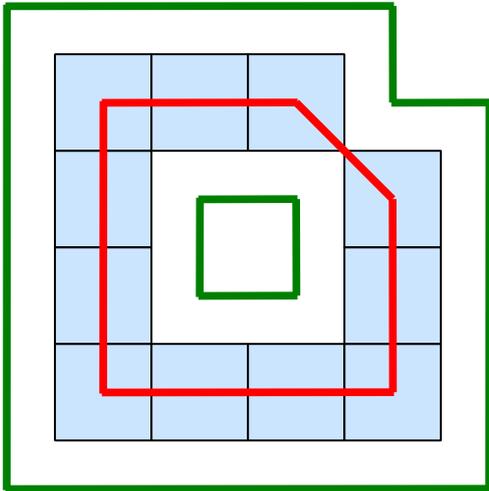
- Need to define one connexity for the (inside) object, and one for the outside



- **Exercise:** possible couples?

# Adjacency couple

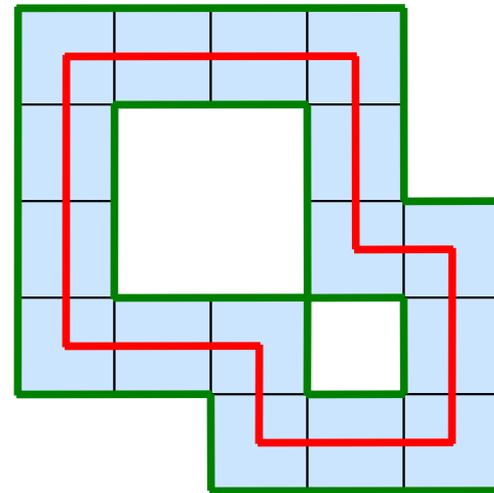
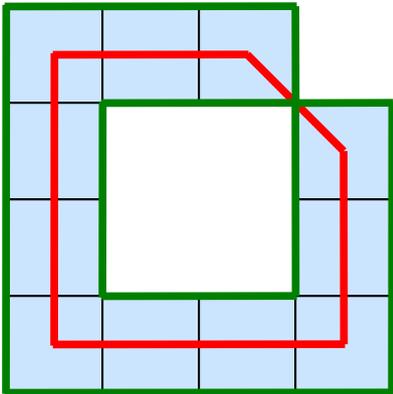
- Need to define one connexity for the (inside) object, and one for the outside



- Possible couples:  $(6, 18)$ ,  $(6, 26)$ ,  $(18, 6)$  and  $(26, 6)$

# Contour

- **Def.:** connected set of cell **faces** between a cell inside the object and a cell outside



- Coherent with Jordan; depends on the chosen adjacency
- Contour of a volume = surface (to display)

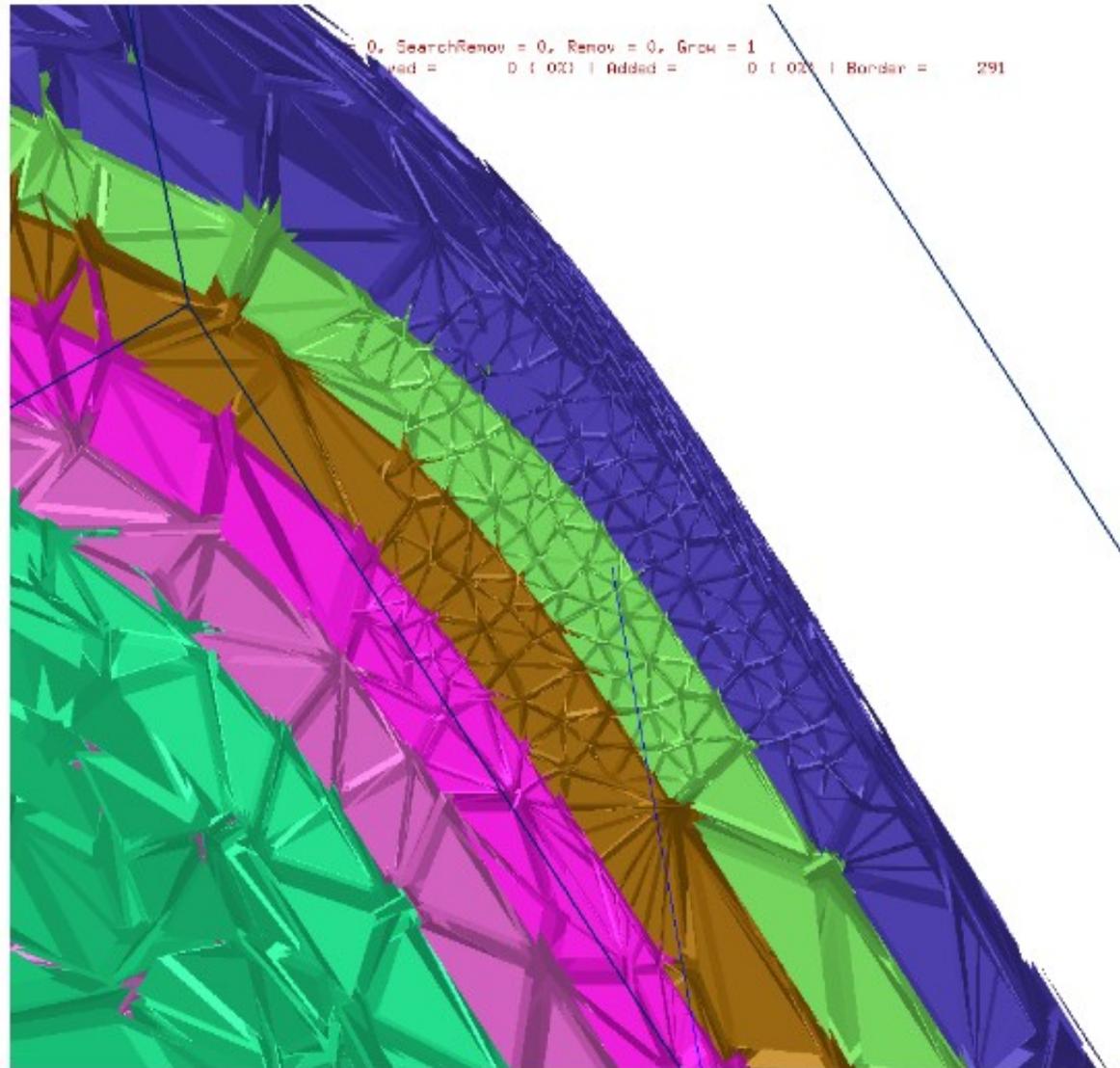
# Discrete geometry

This part was inspired by a course given by  
David Coeurjolly and Isabelle Sivignon  
(CNRS researchers, LIRIS, Lyon)

# Tetrahedra

Not talked about:  
volume modeling  
with tetrahedra  
("tets")

=> finite elements



Courtesy S.Barbier