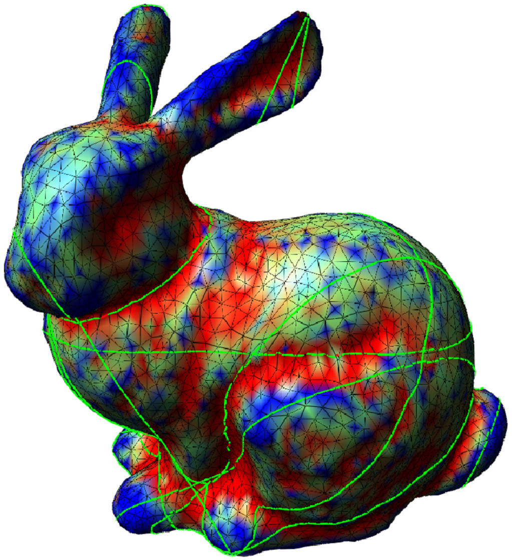
Creating and processing 3D geometry



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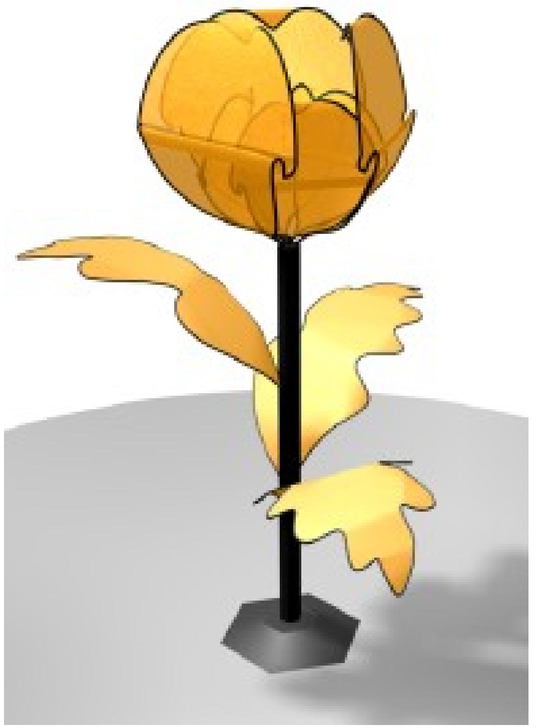


http://evasion.imag.fr/Membres/Franck.Hetroy/Teaching/Geo3D/

Planning (provisional)

Part I – Geometry representations

- Lecture 1 – Oct 9th – FH
  - Introduction to the lectures; point sets, meshes, discrete geometry.
- Lecture 2 – Oct 16th – MPC
  - Parametric curves and surfaces; subdivision surfaces.
- Lecture 3 – Oct 23rd - MPC
  - Implicit surfaces.

Planning (provisional)

Part II – Geometry processing

- Lecture 4 – Nov 6th – FH
  - Discrete differential geometry; mesh smoothing and simplification (paper presentations).
- **Lecture 5 – Nov 13th - CG + FH**
  - Mesh parameterization; **point set filtering and simplification.**
- Lecture 6 – Nov 20th - FH (1h30)
  - Surface reconstruction.

Planning (provisional)

Part III – Interactive modeling

- Lecture 6 – Nov 20th – MPC (1h30)
  - Interactive modeling techniques.
- Lecture 7 – Dec 04th - MPC
  - Deformations; virtual sculpting.
- Lecture 8 – Dec 11th - MPC
  - Sketching; paper presentations.

Books

- This course is inspired from the two following presentations:
  - Mark Pauly, Efficient Simplification of Point-Sampled Surfaces
  - Markus Gross, Spectral Processing of Point-Sampled Geometry

  http://graphics.ethz.ch/publications/tutorials/points/

- See also the books recommended during 1st session

Today's planning

1. Point set simplification
    1. Local surface analysis
    2. Simplification methods
    3. Error measurement
2. Point set filtering
    1. Context
    2. Pipeline

Motivation

- Last week you have seen mesh simplification: why not directly work on the point set ?
  - No connectivity info => faster algorithms ?
  - Scanner acquisition: produces huge point sets
    - Need to be simplified before visualization

Local surface analysis

- ! Point set describing an underlying smooth manifold surface
- Local approximation of the surface
  - Moving Least Squares (MLS)
- Local estimation of tangent plane and curvature
  - Principal Component Analysis (PCA)

Neighborhood

- No explicit connectivity info
- Use spatial proximity
  - Euclidean instead of geodesic distance
- Hyp.: sufficiently dense point set

Least Squares and Weighted Least Squares

- Least Squares Approximation of scattered data:

$$\min_{f \in \prod_m^d} \sum_i \| f(\mathbf{x}_i) - f_i \|^2$$

- f polynomial function
- Weighted Least Squares:

$$\min_{f \in \prod_m^d} \sum_i \theta(\|\bar{\mathbf{x}} - \mathbf{x}_i\|) \, \| f(\mathbf{x}_i) - f_i \|^2$$

- Weight related to distance to a point $\bar{\mathbf{x}}$

Moving Least Squares

- Move x over the parameter domain:

$$f(\mathbf{x}) = f_{\mathbf{x}}(\mathbf{x}), \quad \min_{f_{\mathbf{x}} \in \prod_m^d} \sum_i \theta(\|\mathbf{x} - \mathbf{x}_i\|) \, \|f_{\mathbf{x}}(\mathbf{x}_i) - f_i\|^2$$

- Useful to approximate or interpolate data
- Possible user control (weighting function)
- For more details see the 3-pages report by Andrew Nealen:

  http://www.nealen.com/projects/mls/asapmls.pdf

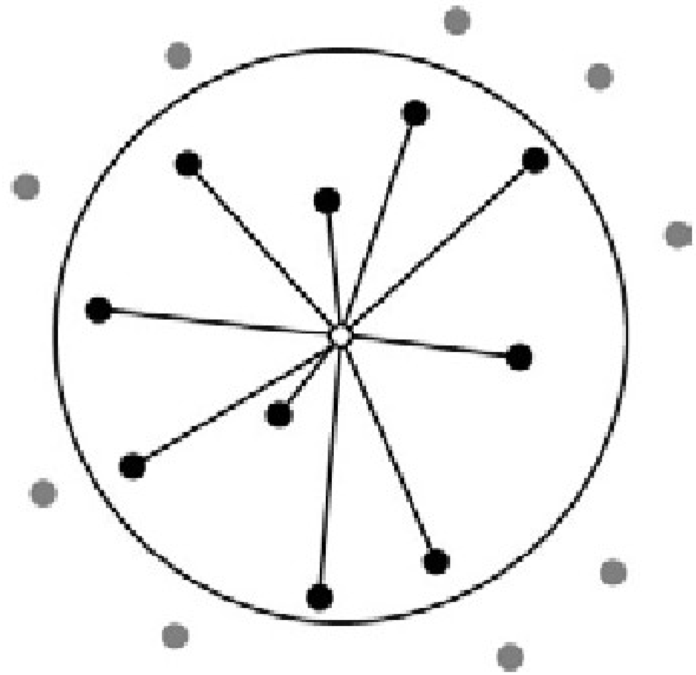
Covariance analysis

- Covariance matrix of local neighborhood N:

$$\mathbf{C} = \begin{bmatrix} \mathbf{p}_{i_1} - \overline{\mathbf{p}} \\ \cdots \\ \mathbf{p}_{i_n} - \overline{\mathbf{p}} \end{bmatrix}^T \cdot \begin{bmatrix} \mathbf{p}_{i_1} - \overline{\mathbf{p}} \\ \cdots \\ \mathbf{p}_{i_n} - \overline{\mathbf{p}} \end{bmatrix}, \qquad i_j \in N$$

- Centroid: $\overline{\mathbf{p}} = \frac{1}{|N|} \sum_{i \in N} \mathbf{p}_i$

K-nearest neighbors



- Can be quickly computed using spatial data structures (octree, BSP-tree, ...)
- Requires isotropic point distribution
- Improvement: angle criterion, local Delaunay

Covariance eigenvectors

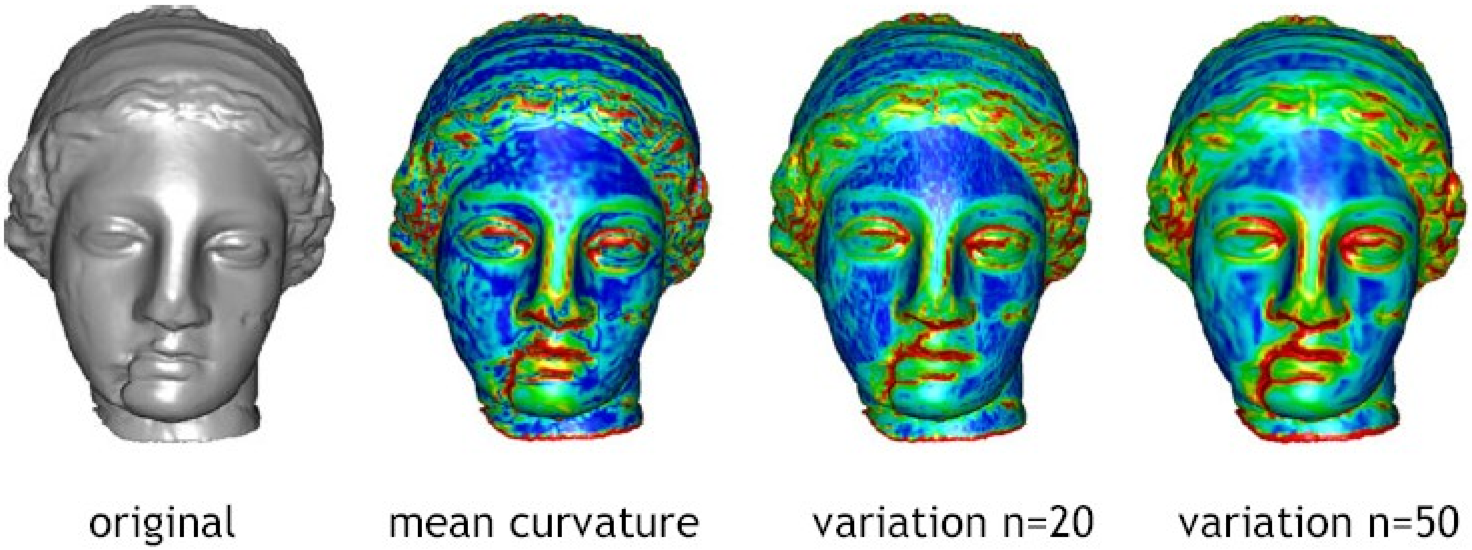
- Eigenvector associated with lowest eigenvalue ~ surface normal
- Eigenvector associated with greatest eigenvalue = axis of greatest variation
- Surface variation:

$$\sigma_n(\mathbf{p}) = \frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_2}, \qquad \lambda_0 \le \lambda_1 \le \lambda_2$$

- = variation along the normal: ~ curvature

Surface variation

- Comparison with mean curvature computed on a mesh representation:



(n = number of neighboring points)

Courtesy M. Pauly

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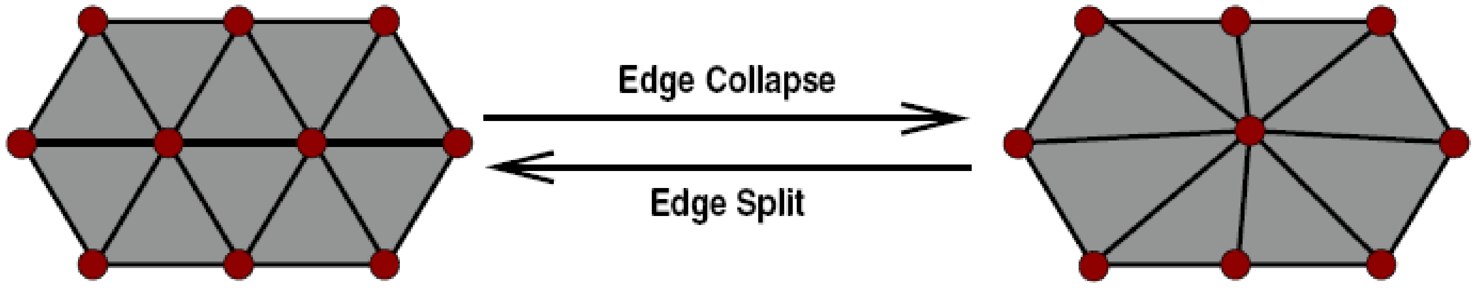
Simplification by subsampling ?

- Simplified point set not necessarily subset of input point set
- Advantage: reduce noise
- Cf. mesh case



Existing approaches

- Mesh simplification approaches:
  - Incremental or hierarchical clustering
  - Iterative simplification (e.g. edge collapse)
  - Particle simulation
- Each has its pros and cons
- Adapt them to point set surfaces

Incremental clustering

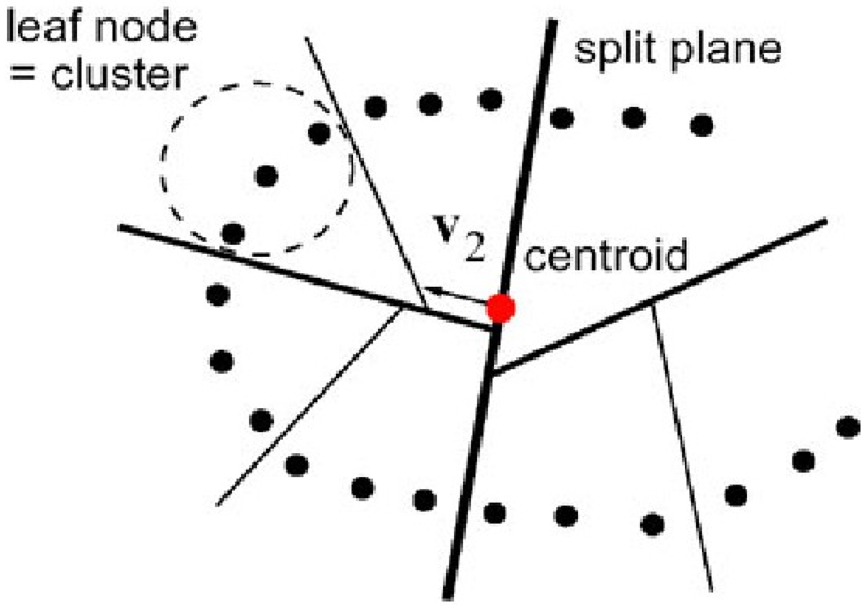
- Region-growing approach
  - Start with random seed point
  - Add nearest points until cluster reaches max size
  - Then choose new seed point from remaining points
- Minimum size threshold to avoid very small clusters
- Cluster size can be controlled by surface variation

Hierarchical clustering

- Top-down approach
- Split the point cloud while:
  - Size > max allowed cluster size
  - Surface variation > max variation
- Leaf nodes correspond to final clusters

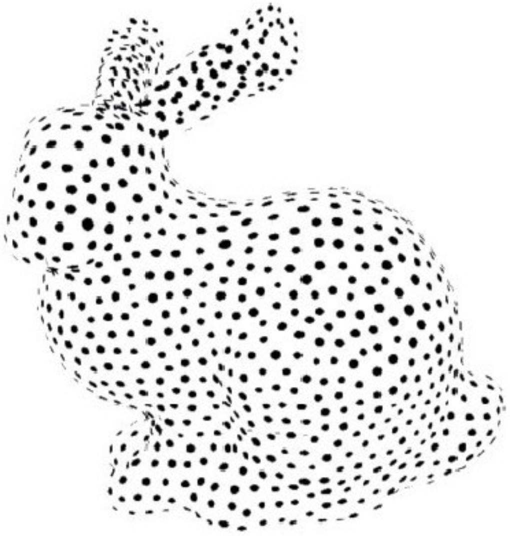
How to split ?

- Split plane defined by
  - Point cloud centroid
  - Axis of greatest variation

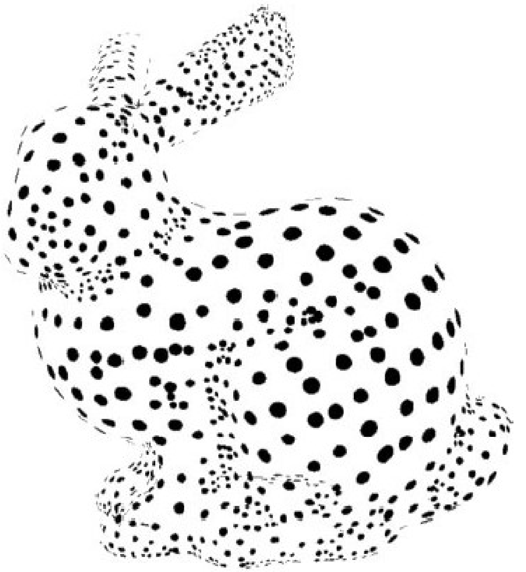


Comparison

- Incremental clustering



- Hierarchical clustering

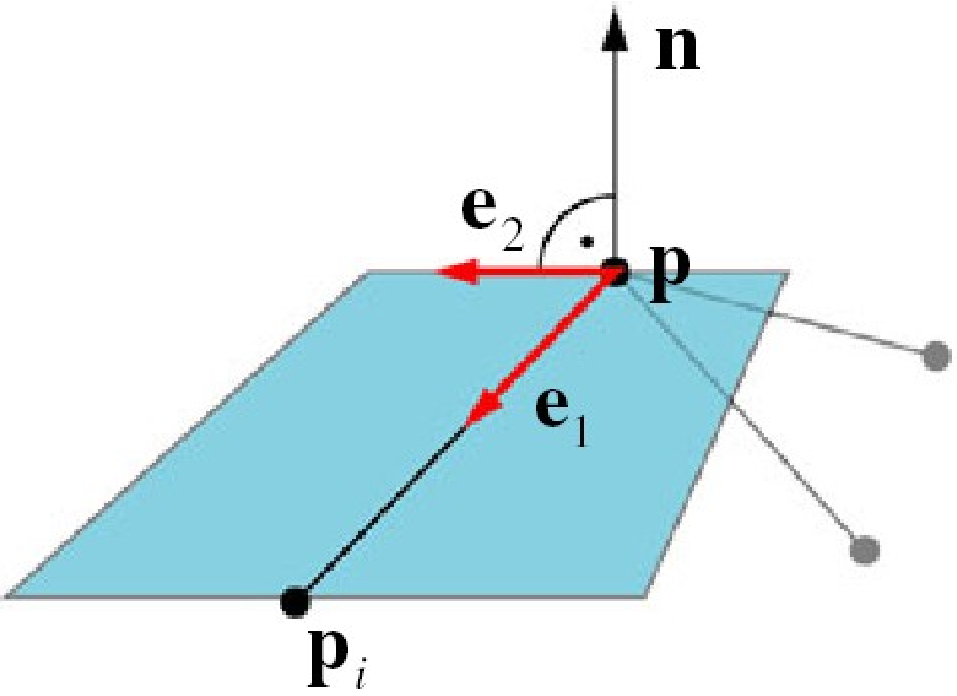


Iterative simplification

- Very similar to Garland and Heckbert's method to simplify a mesh:
  - Iteratively contract point pairs
  - Priority queue
  - Contraction cost and optimal position computed using quadrics
- Main difference: definition of approximating planes

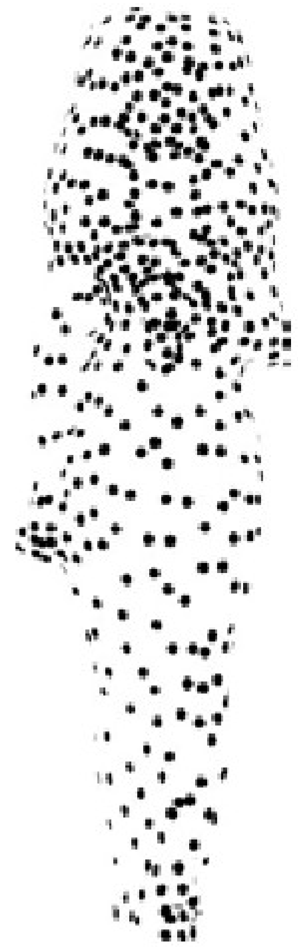
Quadric error

- Squared distance to a set of planes
- Planes defined over edges of neighborhood



Result

original model
(187,664 points)



simplified model
(1,000 points)

Courtesy M. Pauly

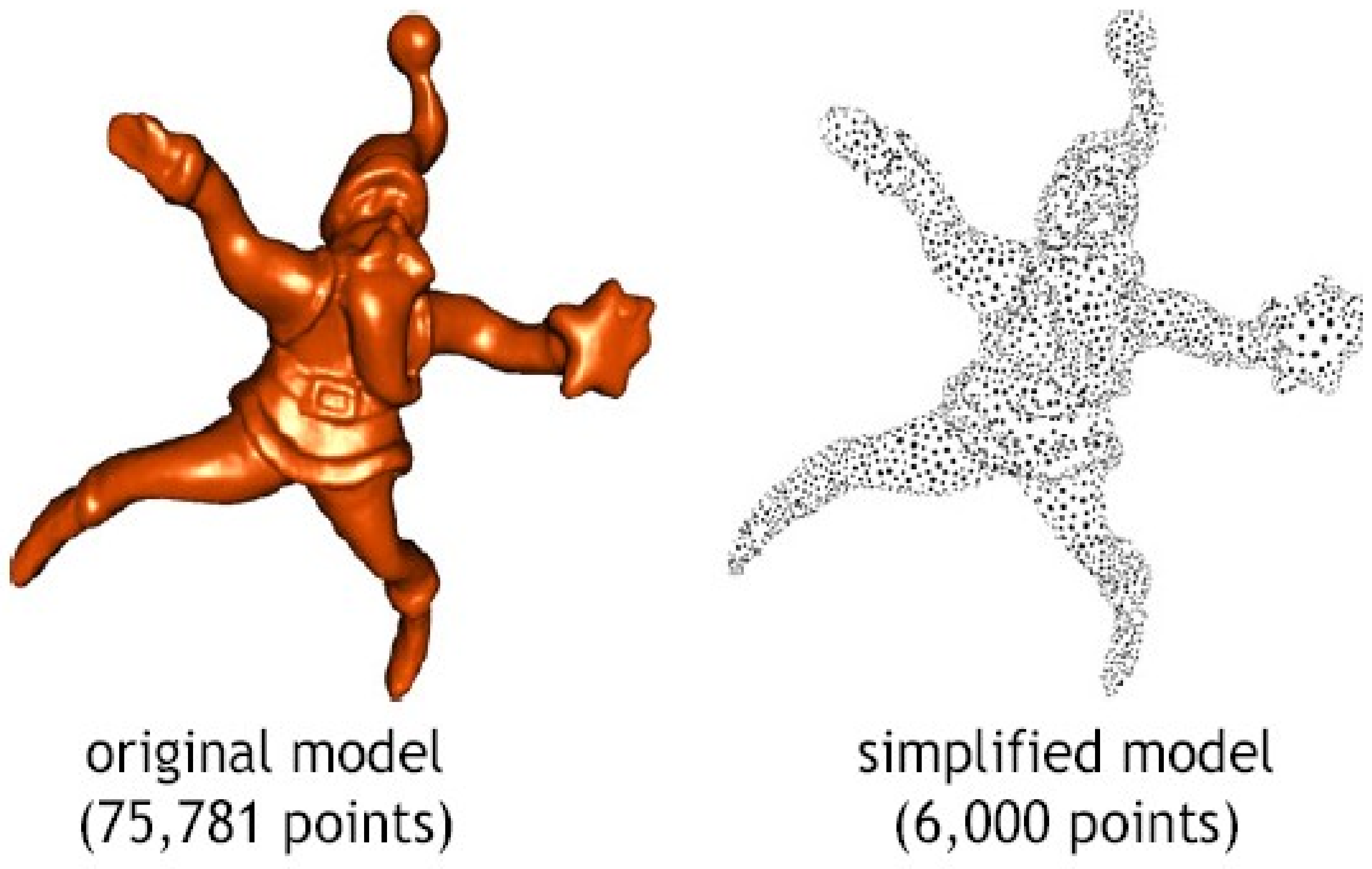
Particle simulation

- Randomly distribute desired number of points (= particles) on the surface
- Particles move on surface according to inter-particle repelling forces
- End when equilibrium is reached

Repelling forces

- Linear repulsion force: $F_i(\mathbf{p}) = k(r - \|\mathbf{p} - \mathbf{p}_i\|) \cdot (\mathbf{p} - \mathbf{p}_i)$
- r = neighborhood radius, k = cst
- Total force = sum of F_i over neighborhood
- Projection:
  - Onto tangent plane of closest point
  - MLS projection at the end

Result



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Comparison

	Efficiency	Surface Error	Control	Implementation
Incremental Clustering	+	-	-	+
Hierarchical Clustering	+	-	-	+
Iterative Simplification	-	+	o	o
Particle Simulation	o	+	+	-

Courtesy M. Pauly

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Distance between 2 point-sampled surfaces

- Maximum error: Hausdorff

$$\Delta_{\max}(S,S') = \max_{\mathbf{q}\in Q} d(\mathbf{q},S')$$

- Mean error: point to surface distance

$$\Delta_{\text{avg}}(S,S') = \frac{1}{|Q|}\sum_{\mathbf{q}\in Q} d(\mathbf{q},S')$$

- Q up-sampled version of the point cloud that describes S
- d(**q**,S') uses MLS projection

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Spectral filtering

- ~ Fourier transform for point sets
- Same idea than Taubin and Desbrun et al. for meshes
- Applications:
  - Fairing (noise removal)
    - Low-pass filter
  - Feature enhancement
    - High-pass filter
  - ...

Fourier transform

- 1D example:

$$X_n = \sum_{k=1}^{N} x_k e^{-j2\pi \frac{nk}{N}}$$

spectral basis function

output signal

input signal

- Benefits:
  - Sound concept of frequency
  - Extensive theory
  - Fast algorithms

Fourier transform

- Requirements:
  - FT defined on Euclidean domain
    - Need of a global parameterization
  - Basis functions ~ eigenvectors of Laplacian operator (cf. Taubin)
    - Regular sampling pattern required for fast evaluation (analytical form)
- Limitation:
  - Lack of local control (basis functions globally defined)

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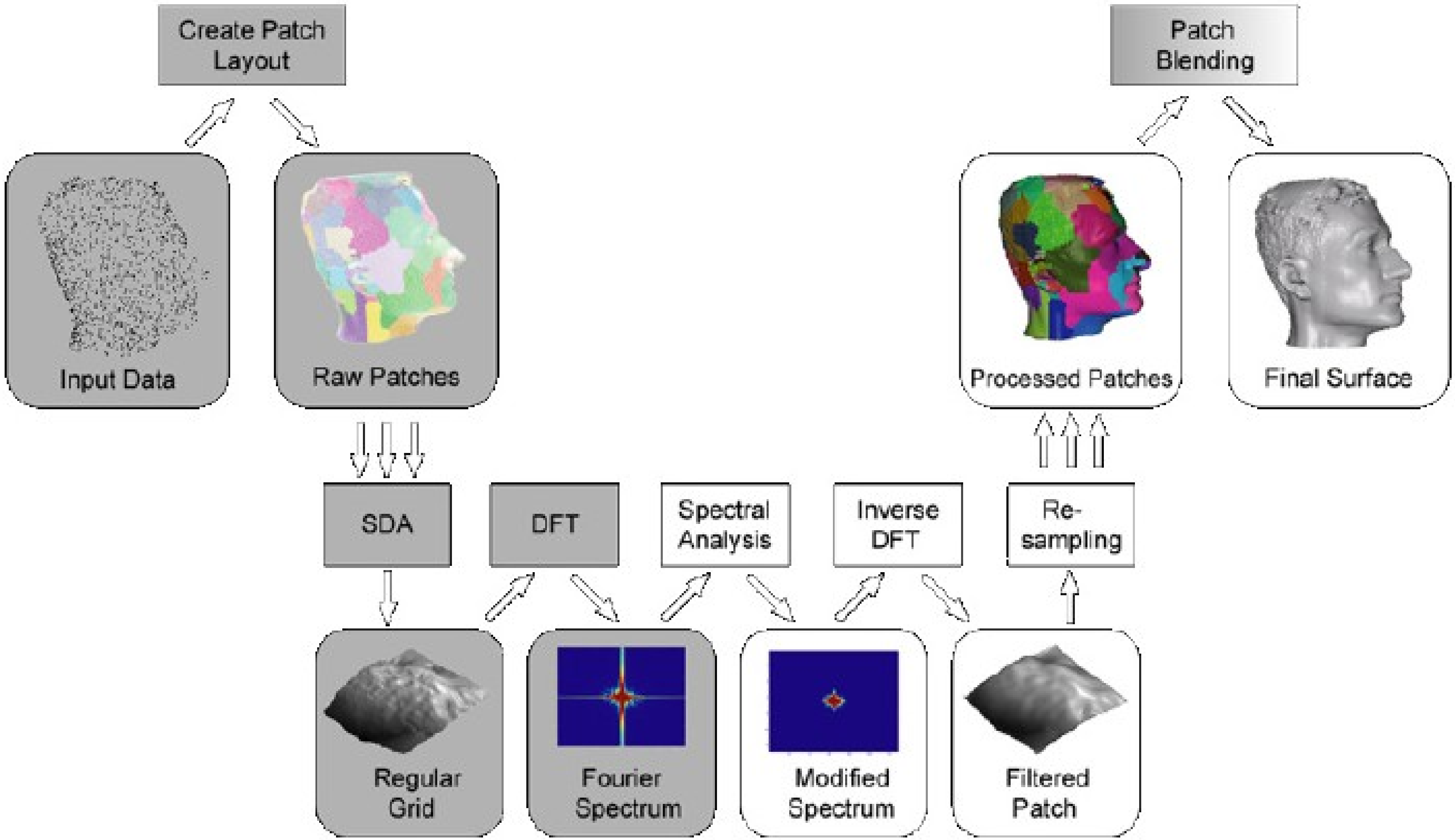
Approach

- To meet the requirements, split the point cloud into patches that:
  - Are parameterized over the unit-square
  - Are re-sampled onto a regular grid
  - Provide sufficient granularity for intended application (local analysis)
- Process each patch individually and blend processed patches

Spectral pipeline



Courtesy M. Gross

The end

- Next week:
  - Surface reconstruction (Franck Hétroy)
  - Interactive modeling techniques (Marie-Paule Cani)
- These slides will be available on the course's webpage:

http://evasion.imag.fr/Membres/Franck.Hetroy/Teaching/Geo3D/