Computer Graphics 2 - Exam

M2R-MoSIG
duration: 3 hours

2009

The suggested durations are just hints. All documents are allowed, but not computers.

1 Rendering (90 minutes)

1.1 Reflectance models

1. (1 point) Give the formula of a BRDF, explain what the terms stand for.
2. (1 point) Give three examples of a BRDF.
3. (3 points) Give the properties a BRDF must follow, with both their physical meaning and their mathematical formulation.

1.2 Global illumination

1. (1 point) Explain the Monte-Carlo integration algorithm, its strengths and its weaknesses.
2. (2 points) Give an example of using the Monte-Carlo integration algorithm for computing global illumination.

1.3 Modeling and expressive rendering

We would like to compute expressive images, based on line extraction, from real 3D models. Using a laser scanner, we get a mesh representation for each input model. Do you think it is useful to convert it to another surface representation (splines, implicit surfaces, ...) ? To compute what kind of lines ? (2 points)

2 Physical articulated bodies (30 minutes)

We want to physically model an articulated body.

1. discuss different possible models for the objects
2. discuss different possible models for their connections.
3. discuss the associated animation algorithms

3 Constrained rigid body dynamics (30 minutes)

The goal of this problem is to model the constrained motion of a system of rigid bodies in contact (without friction). The problem has several questions to guide you through the process. Each question should take a few minutes only.

Notation: In the following, **boldface** denotes vectors and matrices, "." denotes the dot product between two vectors, and "×" denotes the cross product between two vectors.

3.1 Introduction

Let $\mathcal{R}$ denote a rigid body, with center of gravity G, linear velocity $\mathbf{v}(G)$, angular velocity ω, linear acceleration $\mathbf{a}(G)$, angular acceleration α, mass m and inertia tensor $\mathbf{I}$ (remember, $\mathbf{I}$ is a 3×3 symmetric, positive definite matrix).

Recall that the motion of the rigid body is given by (Newton-Euler equations):

$$\begin{aligned} m\mathbf{a}(G) &= \mathbf{f}_e \\ \mathbf{I}\alpha &= \mathbf{\Gamma}_e - \omega \times \mathbf{I}\omega \end{aligned} \tag{1}$$

where $\mathbf{f}_e$ is the *total external force* applied to the rigid body, and $\mathbf{\Gamma}_e$ is the *total external torque* applied to the rigid body.

3.2 Part I: Preliminaries

1. Explain how the Newton-Euler equations above (equation (1)) can be written as a single equation:

$$\mathbf{Ma} = \mathbf{F} + \mathbf{k}. \tag{2}$$

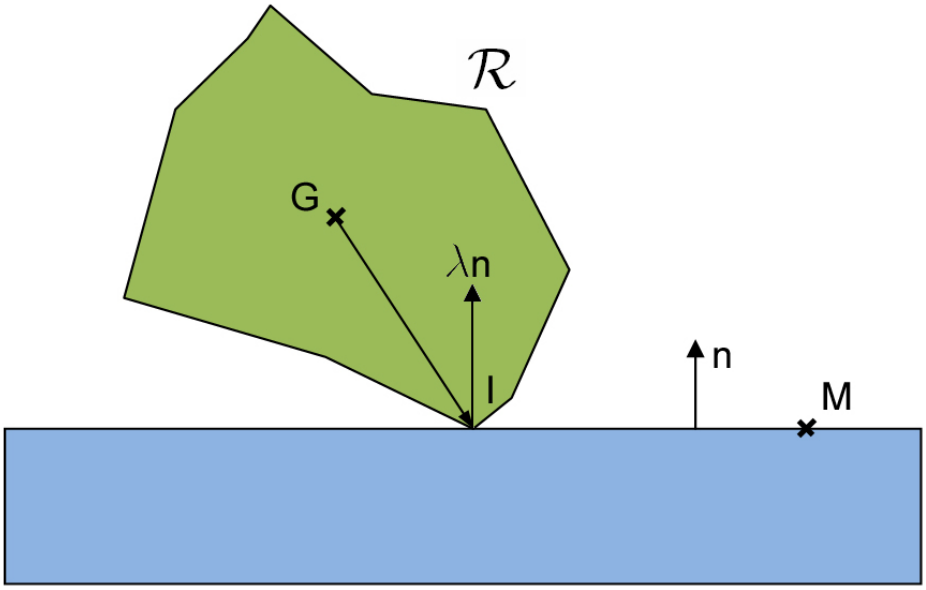

Figure 1: **One rigid body in contact with the environment.**

Precisely, give the dimensions and contents of matrix $\mathbf{M}$ and vectors $\mathbf{a}$, $\mathbf{F}$ and $\mathbf{k}$.

2. Let I denote a fixed point on the rigid body, and let $\mathbf{v}(I)$ denote its velocity. Then, starting from the relationship between $\mathbf{v}(I)$, $\mathbf{v}(G)$ and ω:

$$\mathbf{v}(I) = \mathbf{v}(G) + \omega \times \mathbf{GI}, \tag{3}$$

show that the acceleration $\mathbf{a}(I)$ of I is:

$$\mathbf{a}(I) = \mathbf{a}(G) + \alpha \times \mathbf{GI} + \omega \times (\omega \times \mathbf{GI}). \tag{4}$$

3.3 Part II: One constraint

First, assume that only one rigid body is moving, and that this rigid body in in contact with the (static) environment at point I, as shown in Figure 1.

1. Express the constraint (C) on the *position* of vertex I of the rigid body (*hint*: the contact point I has to stay *above* the plane defined by the point M and the normal $\mathbf{n}$).

2. By taking the derivative of this constraint with respect to time, find the constraint on the velocity $\mathbf{v}(I)$ of the contact point.

3. By taking the derivative of this new constraint with respect to time, find the constraint on the acceleration $\mathbf{a}(I)$ of the contact point.

4. Using the expression of $\mathbf{a}(I)$ determined above (equation (4)), give now the constraint on the linear acceleration $\mathbf{a}(G)$ and angular acceleration α. **Important**: this should be a *linear* constraint written as:

$$\mathbf{a}(G).\mathbf{x} + \alpha.\mathbf{y} \geqslant c, \tag{5}$$

   where you have to determine vectors $\mathbf{x}$ and $\mathbf{y}$, as well as the constant c (*hint*: if $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are three vectors, then $\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}).\mathbf{c})$).

5. Explain how this constraint on the acceleration of the rigid body can be written in a matrix form:

$$\mathbf{J}\mathbf{a} \geqslant \mathbf{c}. \tag{6}$$

   Precisely, give the dimensions and contents of $\mathbf{J}$, $\mathbf{a}$ and $\mathbf{c}$.

6. Let $\mathbf{f}_C = \lambda\mathbf{n}$ denote the contact force (this contact force is proportional to $\mathbf{n}$, since we assume there is no friction). Give (and explain) the constraint on λ.

7. Let $\mathbf{\Gamma}_C = \mathbf{GI} \times \mathbf{f}_C = \lambda(\mathbf{GI} \times \mathbf{n})$ denote the torque caused by the contact force $\mathbf{f}_C$. Show that the concatenated motion equation (2) can now be written:

$$\mathbf{M}\mathbf{a} = \mathbf{F} + \mathbf{k} + \lambda\mathbf{J}^T, \tag{7}$$

   where J^T denotes the transpose of $\mathbf{J}$.

8. Finally, write down the *complementarity constraint*, which corresponds to the fact that *at least* $\mathbf{f}_C$ or $\mathbf{a}(I)$ has to be zero (the contact point either breaks or persists).

3.4 Part III: General case

Explain in a few lines how to generalize this process to the case of n rigid bodies subject to m constraints.

4 The AMC motion capture format (30 minutes)

The AMC motion capture file format is widely used to store data recorded from a motion capture session. Specifications (annex 1) and a short example are provided (annex 2). The goal of this exercise is to identify the mathematical foundation to implement a parser of this format and build information needed to animate a 3D human character.

1. What kind of information are stored in this format ?

2. What kind of 3D object can be animated with it ?

3. Why are there 2 different files (the ASF file and the AMC file) ? What is the benefit ?

4. What mathematical representation is used for rotation matrices in this format ? Identify the keywords involved in the files for rotation matrices.

5. Identify the mathematical relationship in terms of transformation matrices between the left femur joint (lfemur) and the root joint. You will use the following convention for 4x4 transformation matrices :

$$\mathbf{M}(\mathbf{r},\mathbf{t}) = \begin{pmatrix} \mathbf{R}(\mathbf{r}) & \mathbf{t} \\ \mathbf{0} & 1 \end{pmatrix} \tag{8}$$

   where $\mathbf{r}$ is a R^3 vector of rotations parameters producing the 3x3 rotation matrix $\mathbf{R}$ and $\mathbf{t}$ is a R^3 vector of position. You can use pseudocode to formalize the relationship.