Creating & Processing 3D Geometry

Marie-Paule Cani

1. Representations
   - Discrete models: points, meshes, voxels
   - **Smooth boundary: Parametric & Subdivision surfaces**
   - **Smooth volume: Implicit surfaces**
2. Geometry processing
   - Smoothing, simplification, parameterization
3. Creating geometry
   - Reconstruction
   - **Interactive modeling, sculpting, sketching**

1

Drawbacks of Boundary Representations

- Complex shapes with splines ?
  - Branches ?
  - Arbitrary topological genius ?

  Partly solved by subdivision surfaces

- Surrounding a volume?
  - Avoid Klein bottles!
  - Prevent self-intersections

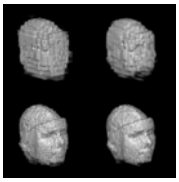
  Make them impossible?

2

Solution
Smooth Volume Representation

Discrete volume

Voxels



Smooth volume

Remains smooth when we zoom in

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Implicit surfaces

Defined by an *Implicit Equation*

$$S = \{ P(x,y,z) \ / \ f(x,y,z) = iso \}$$

- *(f: $R^3 \rightarrow R$)* is the «field function»
- Surface normal : $N = -\nabla f$
- Characterizes a volume! $f(x,y,z) > iso$
  - "***in/out test***" (used for collisions, ray tracing…)
- Smoothness: *S* and *f* have same degree of continuity

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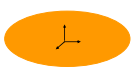
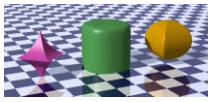
History: Solid Geometry

Volumetric primitives $S = \{ P(x,y,z) \ / \ f(x,y,z) = iso \}$

- Spheres, ellipsoids $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
- Cylinders, cones
- Super-ellipsoids $\frac{x^n}{a^n}+\frac{y^n}{b^n}+\frac{z^n}{c^n}=1$





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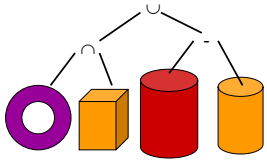
Constructive Solid Geometry

Developed for CAGD

- Solid primitives
- Boolean operators
  - Union (or)
  - Intersection (and)
  - Difference (not)
- Construction tree



Describes the history of construction in a compact, intuitive way

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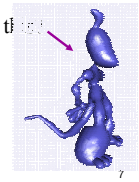
Problem: limited shapes

Free form primitives ?

$S = \{ P(x,y,z) \ / \ f(x,y,z) = iso \}$

f polynomial (algebric surface), or other smooth function

- What should the equation of f be to model t
- How can a user control an implicit shape?
  - Intuitive control
  - Locality
  - Enable deformations



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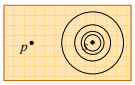
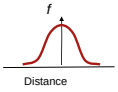
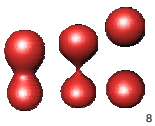
Idea (1982)
Blinn Objects "Blobs"

- Primitive generated by points S
  - f decreasing function of the distance





- Union : $f = max\ (f_1, f_2)$
- Intersection : $f = min\ (f_1, f_2)$
- **Blending** : $f = f_1 + f_2$



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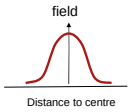
Idea (1982)
Blinn Objects "Blobs"

- Exponential field $f_i = e^{-\frac{d(P-Si)^2}{2}}$
  - \+ Very smooth
  - – No local control
  - – Everything is to be recomputed if a point moves



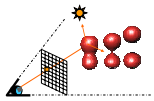
- Extension to blend primitives of different sizes

$$f_i = k_i e^{-\frac{d(P-S_i)^2}{R(S_i)^2}}$$

Displaying implicit surfaces?

Ray Tracing *[Blinn 82]*

- Use dichotomy to compute ray/surface intersections



Later extensions

- Analytical solutions for intersection
- Sphere tracing
  - adapt the step size based on Lipzchisz constants

Several hours for rendering from a single view-point!

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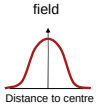
Make implicit surfaces local?
(1985-1990)

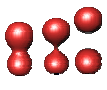
Field function with compact support!

- piece-wise polynomial functions in $d(P,S_i)^2$



- Metaballs *[Nishimura 1985]*
  - *if* $0<d<1/3 \quad f_i = 1 - 3\ d^2$
  - *if* $1/3<d<1 \quad f_i = 3/2\ (1-d^2)$
- Soft Objects *[Wyvill MP W 1986]*

- if $0<d<1 \quad f_i = -4/9\ d^6 + 17/9\ d^4 - 22/9\ d^2 + 1$



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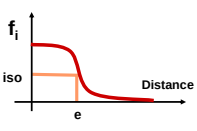
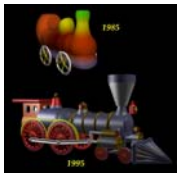
Choice of the field function?

- e gives the thickness of an isolated primitive
- The slope affects the final shape!
- Using $(-f_i)$ instead of f_i carves the shape
  - need of a flat tangent in zero



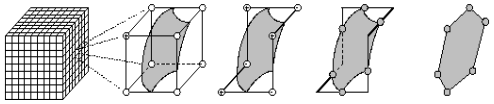
B. Wyvill 85-95



Improving visualization?
(1985-1990)

Marching cubes *[Wyvill MP W 86, Lorensen Cline87]*

- Space grid
- Facetize voxels that cross the surface
- Mesh can be viewed from different viewpoints
- Extension: file to follow the surface



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Improving visualization?
(1985-1990)

Marching cubes *[Bloomenthal 1993-1994]*

- Evaluation of implicit surface tilers [
- An implicit surface polygonizer (paper + code in C)

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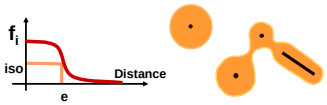
Extensions (1990-2000)

Skeleton-based Implicit Surfaces

Idea: Use any primitive S_i as a skeleton

- $S = \{ P / \sum f_i(P) = iso \}$
- f_i decreasing function of $d(P,S_i)$



Point, segments, disc, cylinder

- Intuitive control, deformation, change of topology

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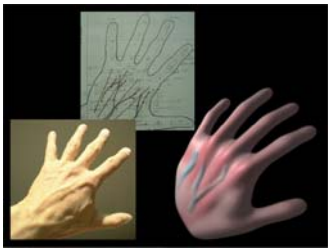
Extensions (1990-2000)

Skeleton-based Implicit Surfaces



J. Bloomenthal 1995

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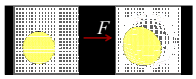
Deforming implicit primitives?

- F space deformation $R^3 \rightarrow R^3$

  Ex: Scale, bend, twist, tapper, etc



- Deformed surface $f_{deformed}(P) = f(F^{-1}(P))$

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Blob tree

- Inspired from CSG trees
  - Blending nodes (+, - , max, min, etc)
  - Unary deformation nodes
- Used for procedural modeling
  - Description file

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Interactive modeling with implicit surfaces? *Fast visualisation*

Particules rendered as splats in the tangent plane

- *[Bloomenthal Wyvill 1991]*
  - Scattered particles projected along the field gradient
- *[Witking Heckbert 1994]*
  - Attraction/repulsion forces
  - Constrained to remain on the surface
  - Split/death of particles

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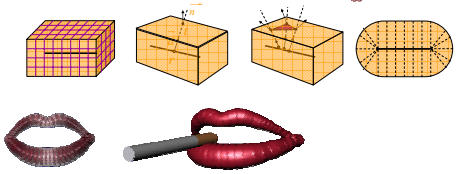
Interactive modeling with implicit surfaces? *Fast visualisation*

- *[Desbrun, Tsingos, Cani 1995]*
  - Sampling of primitive 'territories'
  - Piece-wise polygonization



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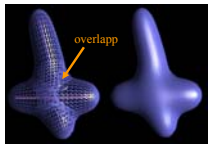
Fast visualization *[Cani Hornus 2001]*

- Overlapping territories

$$\{p \mid \forall j \neq i, f_i(p) + \eta > f_j(p)\}$$

Real-time rendering using OpenGL



- A closed polygonal mesh for each skeleton curve

Old lips New lips

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Modeling complex shapes?

- Branching shapes
- Curved surfaces

Use complex skeletons?

- Intuitive
  - Analogy with medial axis of the shape
- Major problems!
  - Bulges
  - Unwanted blending

Medial axis

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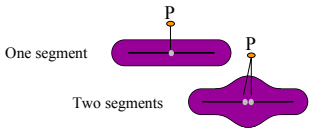
Unwanted Bulges?

Distance surfaces

- Distance to the closest point on S_i
- The shape changes if S_i is divided
- 3D bulge at all joints!



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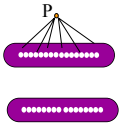
Avoid Unwanted Bulges?

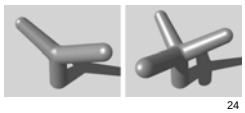
Convolution surfaces *[Bloomenthal Shoemake 91]*

- Integral along Si on point contributions

$$F(S, p) = \int_{s \in S} f(s, p) ds$$

= convolution of the skeleton with a kernel





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Convolution Surfaces

- *[Bloomenthal Shoemake 1991]*
  - Use of Blinn's exponential field
  - Discrete computation of the integral
- *[Sherstyuk 1998-1999]*
  - Closed form (analytical) solutions for specific fields
  - Solutions for segments, triangles, arcs of circles

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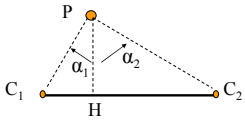
Convolution surfaces
Example of analytical solution

Along a line-segment $f(P) = \int_{r1}^{r2} \frac{1}{r^3}.dr$



$$F(P) = \frac{\sin(\alpha_1) - \sin(\alpha_2)}{d^2(P,H)}$$

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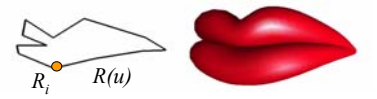
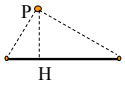
Surfaces of non-constant radius



- R(u) computed from the subdivision mask
- Modified convolution kernel



$$F(P) = \frac{\sin(\alpha_1) - \sin(\alpha_2)}{D^2(P,H)} \quad \text{where} \quad D(P,H) = \sqrt{2}\frac{d(P,H)}{R(H)}$$

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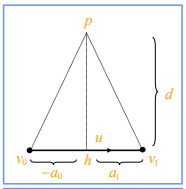
Surfaces of non-constant radius
Exact solution *[Hornus Cani Angelidis 2002]*

- interpolation linéaire de *r*
- convolution de $f(s,p) = \frac{r^2}{d^2(p,s)}c$

$$F(S,p) = \frac{((dC - D^2/d)A + (r_0 - r_1)DB + (a_0 - a_1)C)}{(a_0 - a_1)^2}c$$



$A = \arctan(a_1/d) + \arctan(-a_0/d)$
$B = \log((a_1^2 + d^2)/(a_0^2 + d^2))$
$C = (r_0 - r_1)^2$
$D = r_1 a_1 + r_1 a_0$

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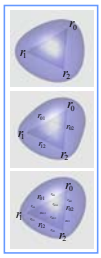
Surfaces of non-constant radius
Not exact on triangles *[Angelidis Cani 2002]*

- Sherstyuk' field [She98]

$$F(S,p) = \int_{s \in S} \frac{1}{(1 + s^2 d^2(p,s))^2} dS$$

- Practical solution to varying radius

$$\frac{r_0 + r_1 + r_2}{3} F(S,p)$$



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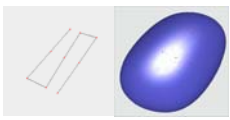
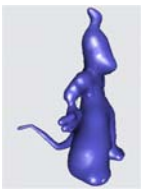
Unwanded Blending problem

- Primitives blend according to their distance!





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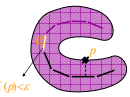
Solutions to Unwanded Blending

Idea: "blending graph" expressing the shape's topology

- *[Guy Wyvill 1995]*
  Find the main primitive
  Add its immediate neighbours
- *[Cani Hornus 2001]*
  - blend until the contribution is small enough



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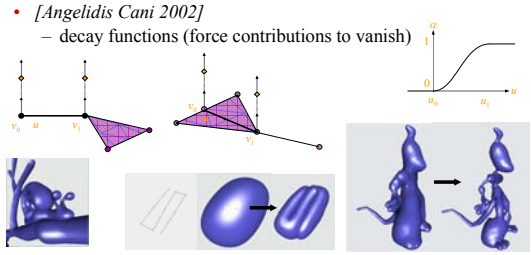
Solutions to Unwanded Blending

Idea: "blending graph" expressing the shape's topology

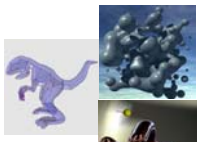
- *[Angelidis Cani 2002]*
  - decay functions (force contributions to vanish)



Modeling complex shapes
Best implicit representation?

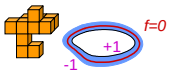
Constructive representation (tree)

- Lots of very simple primitives?
  objects breaking into pieces
- Fewer, complex skeletons?
  skeleton-based deformations



Discrete field, with smooth interpolation?

- Constant time field queries
- No limitation of complexity!



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Multi-resolution Implicit Surfaces?

This means:

- Multi-resolution field function
- To combine with an adaptive polygonization

Discrete representations (sculpting…)

- Multi-grids *[Frisken Perry 00, Ferley Cani Gascuel 2001]*

Constructive approaches

- Multi-resolution skeletons?

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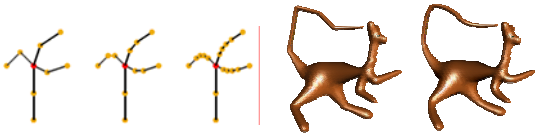
Multi-resolution Implicit Surfaces?

Subdivision curves & surfaces as implicit skeletons

- Curves only: refine branches *[Cani Hornus 2001]*



- Subdivision curves & surfaces *[Angelidis Cani 2002]*

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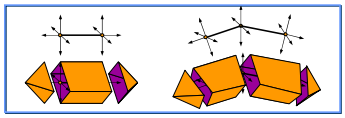
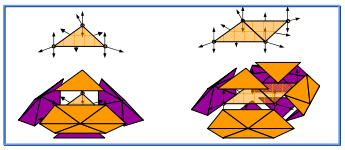
Subdivision curves & surfaces as skeletons
Adaptive polygonization

- segments



- triangles



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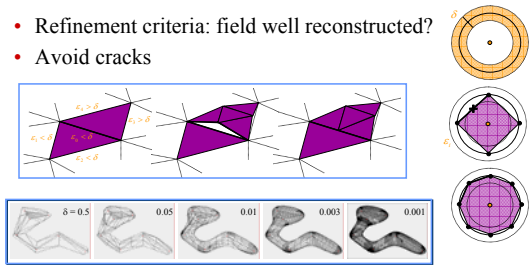
Subdivision curves & surfaces as skeletons

Adaptive polygonization

- Refinement criteria: field well reconstructed?
- Avoid cracks



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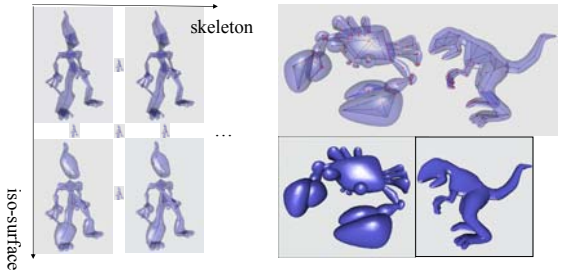
Subdivision curves & surfaces as skeletons

Results [Angelidis Cani 2002]



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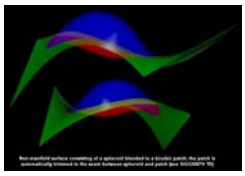
Advanced bibliography

Polygonization of Non-Manifold Implicit Surfaces

[Bloomenthal Ferguson SIGGRAPH 1995]

- Implicit surfaces can be used to model open shapes!
- Non-manifold blends between a volume and an open surface
- Definition & polygonization



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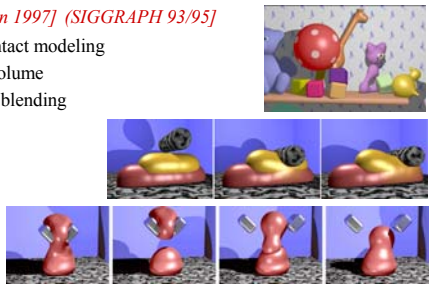
Advanced bibliography

Animation of Deformable Models Using Implicit Surfaces

[Cani Desbrun 1997] (SIGGRAPH 93/95]

- Precise contact modeling
- Constant volume
- Controlled blending

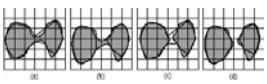


Advanced bibliography

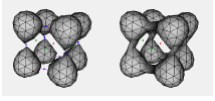
Guaranteeing the Topology of an Implicit Surface Polygonization

[Stander Hart SIGGRAPH 1997]

- Morse theory used to track critical points
- Garanteed correct topology!



Marching cube correct on (a) (d)
but fails on translated shapes (b) & (c)



Tracking critical points

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Advanced bibliography

Adaptive Implicit Surface Polygonization Marching Triangles

[Galin Akkouche, Computer Graphics Forum 2001]

- Good quality meshing of implicit surfaces
  - marching triangles, instead of marching cubes
  - Size adapted to local curvature
  - Use in an interactive modeling system

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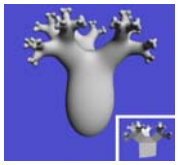
Advanced bibliography

Modeling with Implicit Surfaces that Interpolate

[Turk, O'Brien, SIGGRAPH 2002]

- Introduction of *variational implicit surfaces*
  - Defined by solving a linear system of position constraints
  - Now widely used for reconstructing/re-sampling point sets



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Open problems in implicit modeling??

- Interactive techniques
  - Sculpting versus sketching metaphors
  - Interactive deformations, constant volume
- General method for real-time rendering?
- Levels of details
  - Extend subdivision to multi-resolution
  - Adaptive topology for the skeleton ?
- Texture mapping?

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