Creating & Processing 3D Geometry

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1. Representations
   - Discrete models: points, meshes, voxels
   - **Smooth boundary: Parametric & Subdivision surfaces**
   - **Smooth volume: Implicit surfaces**
2. Geometry processing
   - Smoothing, simplification, parameterization
3. Creating geometry
   - Reconstruction
   - **Interactive modeling, sculpting, sketching**

1

Choice of a representation?

Notion of 'geometric model'
- Mathematical description of a virtual object (enumeration/equation of its surface/volume)

How should we represent this object…
- To get something smooth where needed ?
- To have some real-time display ?
- To save memory ?
- To ease subsequent deformations?

2

Why do we need Smooth Surfaces ?

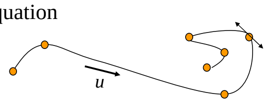
Meshes
- Explicit enumeration of faces
- Many required to be smooth!
- Smooth deformation???

Smooth surfaces
- Compact representation
- Will remain smooth
  - After zooming
  - After any deformation!

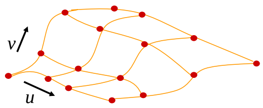
3

Parametric curves and surfaces

Defined by a parametric equation
- Curve: *C(u)*
- Surface: *S(u,v)*



Advantages
- Easy to compute point
- Easy to discretize
- Parametrization



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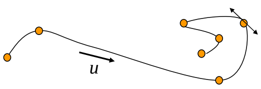
Parametric curves: Splines

Motivations : interpolate/approximate points P_k

- Easier too give a finite number of "control points"
- The curve should be smooth in between

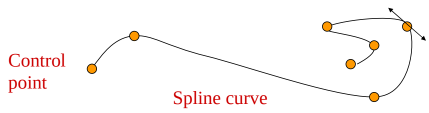


Why not polynomials? Which degree do we need?

5

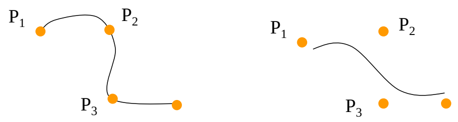
Spline curves

- Defined from control point
- Local control
  - Joints between polynomial curve segments
  - degree 3, C^1 or C^2 continuity



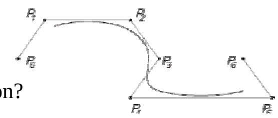
6

Interpolation vs. Approximation



7

Splines curves



Mathematical formulation?

- Curve points = linear combination of control points

$$C(u) = \Sigma F_k(u)\, P_k$$

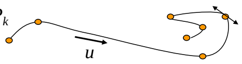
  – Curve's degree of continuity = degree of continuity of F_k

Desirable properties for the "influence functions" F_k?

8

Properties of influence functions? 1. Affine invariance

$C(u) = \Sigma F_k(u)\, P_k$



Invariance to affine transformations?

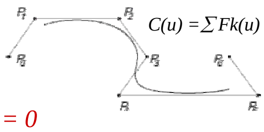
  – Same shape if control points are translated, rotated, scaled

$$\Sigma F_k(u) = 1$$

- Influence coefficients are barycentric coordinates
- Prop: barycentric invariance too. Application to morphing

9

Properties of influence functions? 2. Convex hull



$C(u) = \Sigma Fk(u)\ Pk$

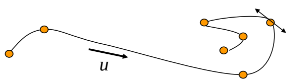
Convex hull: $F_k(u) >= 0$

Curve points are barycenters

- Draw a normal, positive curve which interpolates
- Can it be smooth?

10

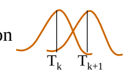
Properties of influence functions? 3. Variance reduction



No unwanted oscillation?

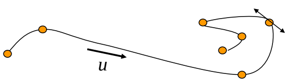
Nb intersections curve / plane <= control polygon / plane

  – A single maximum for each influence function



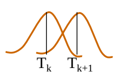
11

Properties of influence functions? 4. Locality



Local control on the curve?

  – easier modeling, avoids re-computation



Choose Fk with local support

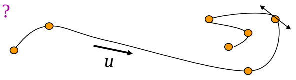
  – Zero and zero derivatives outside an influence region
  – Are they really polynomials?

12

Properties of influence functions?
5. Continuity: parametric / geometric

- Parametric continuity C1, C2, etc
  - Easy to check
  - Important if the curve defines a trajectory!

Ex: $q(u) = (2u,u)$, $r(t)=(4t+2, 2t+1)$.
Continuity at $J=q(1)=r(0)$?

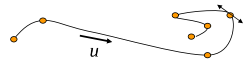


- Geometric continuity G1, G2, etc

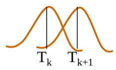
13

Splines curves
Summary of desirable properties

$C(t) = \sum F_k(t)\, P_k$,



- Interpolation & approximation
  - Affine invariance $\sum F_k(t) = 1$
  - locality $F_k(t)$ with compact support
  - Parametric or geometric continuity
- Approximation
  - Convex envelop $F_k(t) \geq 0$
  - Variance reduction: no unwanted oscillation



14

Splines curves
Most important models

- Interpolation
  - Hermite curves C[1], cannot be local if C[2]
  - Cardinal spline (Catmull Rom)
- Approximation
  - Bézier curves
  - Uniform, cubic B-spline (unique definition, subdivision)
  - Generalization to NURBS

15

Cardinal Spline, with tension=0.5

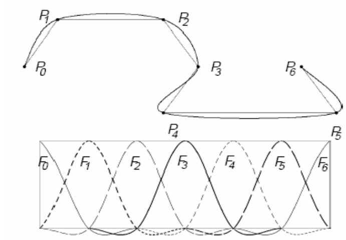


Figure 2: *Catmull-Rom spline curve*

16

Uniform, cubic Bspline

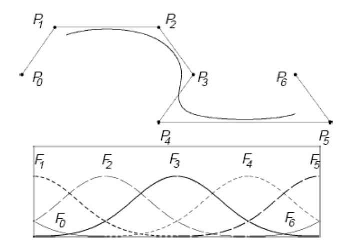


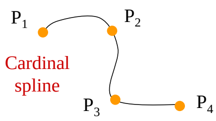
Figure 1: *Uniform B-spline curve*

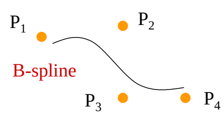
17

Cubic splines: matrix equation

$Q_i\,(u) = (u^3\, u^2\, u\ 1)\, M_{spline}\, [P_{i-1}\, P_i\, P_{i+1}\, P_{i+2}]^{\,t}$

$$M_{Catmull} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \qquad M_{Bspline} = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$





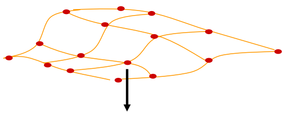
18

Splines surfaces

« Tensor product »: product of spline curves in *u* and *v*

$Q_{i,j}(u, v) = (u^3\ u^2\ u\ 1)\ M\ [P_{i,j}]\ M^t\ (v^3\ v^2\ v\ 1)$

– Smooth surface?
– Convert to meshes?
– Locallity?

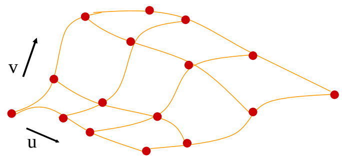


19

Splines surfaces

- Expression with *separable* influence functions!

$Q_{i,j}(u, v) = \sum B_i(u)\, B_j(v)\, P_{ij}$

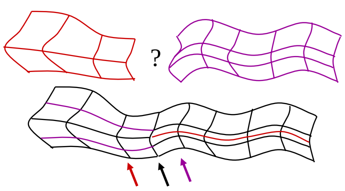


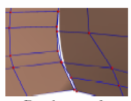
Historic example

20

Can splines represent complex shapes?

- Fitting 2 surfaces : same number of control points





a. B-spline surfaces

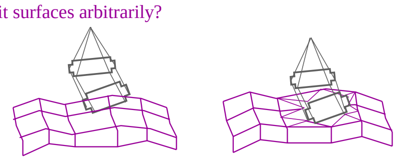
21

Can splines represent Complex Shapes?

Closed surfaces can be modeled

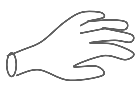
- Generalized cylinder: duplicate rows of control points
- Closed extremity: degenerate surface!

Can we fit surfaces arbitrarily?



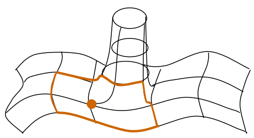
22

Can splines represent Complex Shapes?



Branches ?

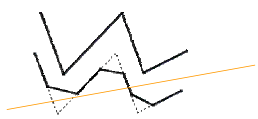
- 5 sided patch ?
- joint between 5 patches ?

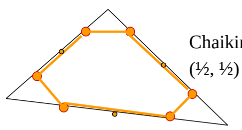


23

Subdivision Curves & Surfaces

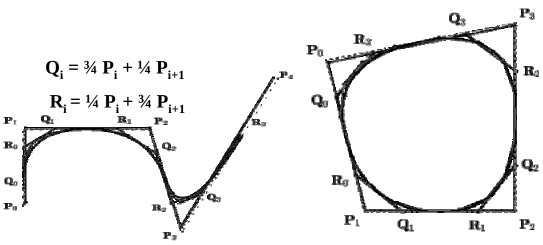
- Start with a control polygon or mesh
- progressive refinement rule (similar to B-spline)
- *Smooth?* use variance reduction!
  – *"corner cutting"*





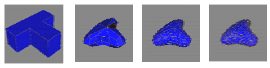
24

How Chaikin's algorithm works?

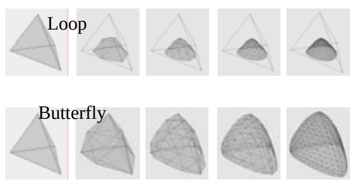


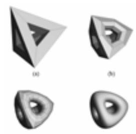
25

Subdivision Surfaces



- Topology defined by the control polygon
- Progressive refinement (interpolation or approximation)

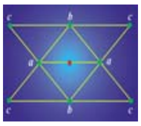




Catmull-Clark
= B-spline at regular vertices

26

Example : Butterfly Subdivision Surface



- **Interpolate**
- **Triangular**
- Uniform & Stationary
- Vertex insertion (primal)
- 8-point

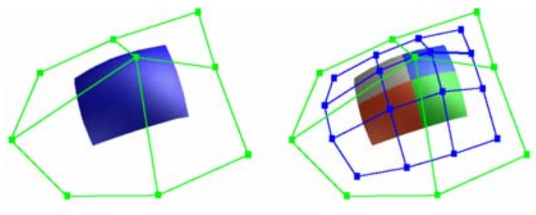
a = ½, b = 1/8 + 2w, c = -1/16 – w

w is a tension parameter

w = –1/16 => surface isn't smooth

27

Example: Doo-Sabin
Works on quadrangles; Approximates



(a) Original control mesh. (b) Control mesh after one subdivision.

28

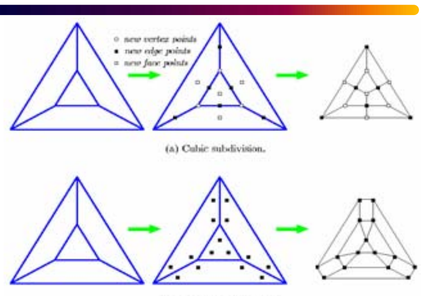
Comparison

Catmull-Clark

(primal)

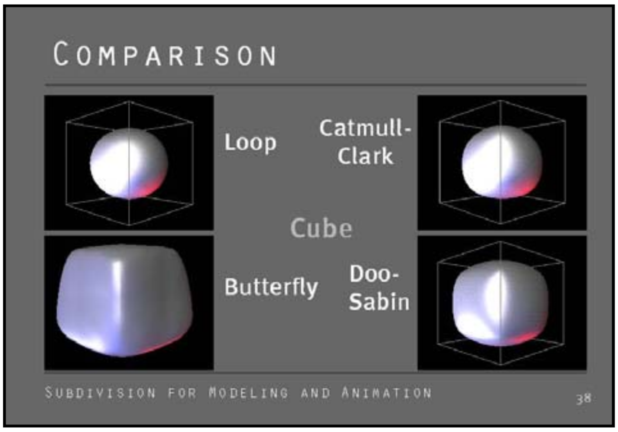
Doo-Sabin

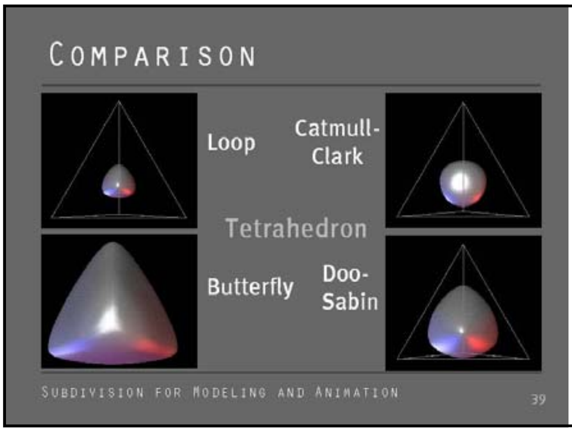
(dual)



(a) Cubic subdivision.

(b) Quadratic subdivision.





At curve point / regular surface vertices
Splines as limit of subdivision schemes

Quadratic uniform B-spline curve	←→	Chaikin
Quadratic uniform B-spline surface	←→	Doo-Sabin
Cubic uniform B-spline surface	←→	Catmull-Clark
Quartic uniform box splines	←→	Loop

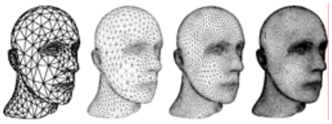
32

Subdivision Surfaces

Benefits
- Arbitrary topology & geometry (branching)
- Approximation at several levels of detail (LODs)

Drawback: No parameterization, some unexpected results

Loop

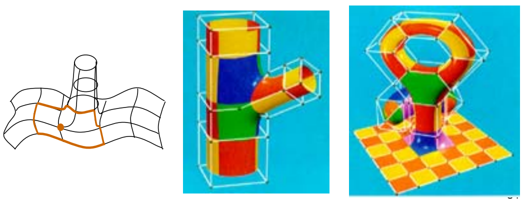


Extension to multi-resolution surfaces : Based on wavelets theory

33

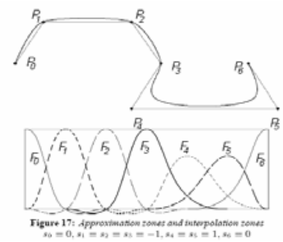
Advanced bibliography
1. Generalized B-spline Surfaces of Arbitrary Topology

[Charles Loop & Tony DeRose, SIGGRAPH 1990]
- n-sided generalization of Bézier surfaces: “Spatches”



Advanced bibliography
2. Xsplines [Blanc, Schilck SIGGRAPH 1995]

Approximation & interpolation in the same model

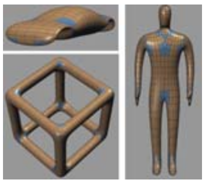


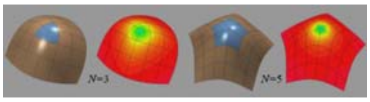
35

Advanced bibliography
3. Exact Evaluation of Catmull-Clark Subdivision

[Jos Stam, Siggraph 98]
Analytic evaluation of
surface points and derivatives

- Even near irregular vertices,
- At arbitrary parameter values!

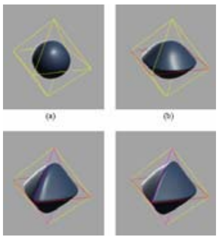




36

Advanced bibliography
4. Subdivision Surfaces in Character Animation

[Tony DeRose, Michael Kass, Tien Truong, Siggraph 98]



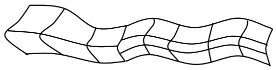
Keeping some sharp creases where needed

37

Advanced bibliography
5. T-splines & T-NURCCs [Sederberg et. Al., Siggraph 2003]

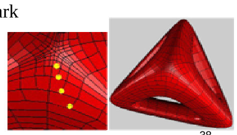
T-splines d^3, C^2: superset of NURBS, enable T junctions!

- Local lines of control points
- Eases merging



T-NURCCs: Non-Uniform Rational Catmull-Clark Surfaces with T-junctions

- superset of T-splines & Catmull-Clark
- enable local refinement
- same limit surface.
- C2 except at extraordinary points.



38

Comment représenter la géométrie ?

- Représentations par bord / surfaciques / paramétriques
  - **Polygones (surfaces discrètes)**
  - **Surfaces splines**
  - **Surfaces de subdivision**, *surfaces multi-résolution*
- Représentations volumiques / implicites
  - **Voxels (volumes discrets)**
  - CSG (Constructive Solid Geometry)
  - Surfaces implicites

Adapter le choix aux besoins de l'animation et du rendu !

39