Creating and processing 3D geometry

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http://evasion.imag.fr/Membres/Franck.Hetroy/Teaching/Geo3D/
We want to represent objects
- Real objects
- Virtual/created objects

Several ways for virtual object creation
- Interactive by graphists
- Automatic from real data
  - 3D scanner, medical angiography, ...
- Procedural (on the fly)
  - Complex scenes, terrain, ...

Different uses
- Display, animation, physical simulation, ...
Course overview

1. Objects representations
   - Volume/surface, implicit/explicit, ...
Course overview

1. Objects representations
   - Volume/surface, implicit/explicit, ...

2. Geometry processing
   - Simplify, smooth, ...

Interactive multiresolution surface exploration
Course overview

1. Objects representations
   - Volume/surface, implicit/explicit, ...

2. Geometry processing
   - Simplify, smooth, ...

3. Virtual object creation
   - Surface reconstruction, interactive modeling

Shape modeling by sketching
Part I – Geometry representations

- **Lecture 1 – Oct 9th – FH**
  - Introduction to the lectures; point sets, meshes, discrete geometry.

- **Lecture 2 – Oct 16th – MPC**
  - Parametric curves and surfaces; subdivision surfaces.

- **Lecture 3 – Oct 23rd - MPC**
  - Implicit surfaces.
Planning (provisional)

Part II – Geometry processing

- **Lecture 4 – Nov 6th – FH**
  - Discrete differential geometry; mesh smoothing and simplification (*paper presentations*).

- **Lecture 5 – Nov 13th - CG + FH**
  - Mesh parameterization; point set filtering and simplification.

- **Lecture 6 – Nov 20th - FH (1h30)**
  - Surface reconstruction.
Planning (provisional)

Part III – Interactive modeling

● Lecture 6 – Nov 20th – MPC (1h30)
  – Interactive modeling techniques.

● Lecture 7 – Dec 04th - MPC
  – Deformations; virtual sculpting.

● Lecture 8 – Dec 11th - MPC
  – Sketching; paper presentations.
Books

For my part of the course:

• M. Botsch et al., “Geometric Modeling Based on Polygonal Meshes”, SIGGRAPH 2007 Course Notes.

http://graphics.ethz.ch/~mbotsch/publications/sg07-course.pdf

!!! Also test the source code:

http://graphics.ethz.ch/~mbotsch/publications/meshcourse07_code.tgz
Books

For Marie-Paule's part of the course:

  - Geometry representations

  - Interactive modeling
Factual information

- 9h-10h30 + 10h45-12h15
- This room (008)
- Mark:
  - 1 final written exam (1/2)
  - Geometry processing paper presentation + demo (1/4)
  - Interactive modeling paper presentation (1/4)
Geometry processing paper

• By groups of 2 students
• You are asked to:
  – Choose a paper among the proposed ones
  – Prepare a short presentation (10 minutes + 5 minutes for questions), which includes a demo
• PDF files and basic interface and data structures on the course's webpage:
  http://evasion.imag.fr/Membres/Franck.Hetroy/Teaching/Geo3D
Proposed papers

• Two topics
  – Mesh smoothing (3 papers)
  – Mesh simplification (3 papers)

• Send an e-mail to Franck.Hetry@imag.fr when chosen

• Presentations: November, 6th
Mesh smoothing papers


Mesh simplification papers


1. Introduction to the course
2. Geometry representations: introduction
3. Point sets
4. Meshes
5. Discrete geometry
Today's planning

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Geometry representations

- **Today:**
  - Point sets
  - (Flat) Meshes
  - Voxels

- **Next week:**
  - Parametric curves and surfaces (splines, ...)
  - Multiresolution meshes

- **In two weeks:**
  - Implicit surfaces
Geometry representations

- A good introduction to all these representations is in chapter 2 of Botsch et al.'s book
  - Parametric/explicit surfaces: splines, subdivision surfaces, triangle meshes
  - Implicit surfaces
  - Conversion from one rep. to the other
  - Only about surfaces: point sets volumetric rep.
Why not one good representation?

- Multiple applications, different constraints
  - Powerful rep.
    - To handle a large class of objects
    - To create complex objects from simple ones
  - Intuitive rep.
    - To edit the model
    - To animate some parts of it
  - Efficient rep.
    - Memory cost
    - Display/process time cost
Classification: a proposal

- **Non structured rep.**
  - Point set
  - Polygon soup
- **Surface rep.**
  - Mesh
  - Parametric
  - Subdivision
  - Implicit
- **Volumetric rep.**
  - Voxel line/plane/set
  - Octree
  - CSG
- **Procedural rep.**
  - Fractal
  - Grammar/L-system
  - Particle system
- **Image-based rep.**
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Point sets

• Result of scanner acquisition
• Also image-based modeling
• Main advantages:
  – “Natural” representation
  – Simple and cheap to display
• Main drawbacks:
  – No connectivity info: underlying shape = ?
  – Tedious to edit
Too simple?

• If nb of points too low: holes

• However:
  – Currently scanned models have up to several millions points
  – Mesh reconstruction is then time-consuming
  – Memory to store the mesh also a problem (number of faces \( \sim 2 \times \) number of points)
  – Each face projects onto only one or two pixels!

• That is why surface representation by a point set is more and more used and studied
Point set representation

- Points are **samples** of the underlying surface
- 1 point corresponds to 1 **surfel** (surface element)
  - Position
  - Color
  - Normal
  - Radius
- Surfel = 2D !

Courtesy M. Zwicker
Surfel

- Surfels are designed mostly for rendering
- Advantage: no mesh reconstruction necessary
  – Time saving
- No surface connectivity information

Courtesy M. Zwicker
Point set

Forward warping

Shading

Visibility

Framebuffer

Image reconstruction

Credit: M. Zwicker 2002
Forward warping and shading

- Forward warping = perspective projection of each point in the point cloud
- Similar to projection of triangle vertices (mesh case)
- Shading:
  - Per point
  - Conventional models for shading (Phong, Torrance-Sparrow, reflections, etc.)
  - Cf. rendering course
Visibility and image reconstruction

- Performed simultaneously
- Discard points that are occluded from the current viewpoint
- Reconstruct continuous surfaces from projected points
Image reconstruction

- **Goal:** avoid holes in the image of the surface
- Use surfel radius to cover the surface
- More during the rendering course
Point set processing

• Some work on:
  – Simplification
  – Filtering
  – Decomposition, resampling

• Still lack of robust mathematical theory
  – Cf. the mesh case (session 4)

• (Possible) Topic of the session 5 of this course
Surface approximation

- Almost all other surface representations are based on points
  - Meshes
  - Parametric rep. (splines)
  - Implicit rep.
- A projection-based surface definition is also possible
  - Local polynomial P around each point
  - Project P(0) onto a local reference plane
Books

- M. Alexa et al., “Point-Based Computer Graphics”, SIGGRAPH 2004 Course Notes
  http://graphics.ethz.ch/publications/tutorials/points/


- See also works by Mark Alexa (TU Berlin), Markus Gross et al. (ETH Zürich), Gaël Guennebaud et al. (IRIT Toulouse, now ETH Zürich)
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Meshes

- Mesh = (V,E,F)
  - V = set of vertices
  - E = set of edges
  - F = connected set of (planar) faces
- Not connected = polygon soup
- Faces can be
  - Triangles
  - Planar quads
  - Any planar, convex polygon
Meshes

- Main **advantage**: easy display
- Main **drawback**: tedious to edit
- Represent continuous piecewise linear surfaces
- Encode
  - (Approximate) **geometry**
    - OK for planar shapes (CAD)
    - Bad for curved shapes
  - **Topology** (see 2 slides after)
2-Manifold

- **Def.**: each vertex has a neighborhood on $M$ homeomorphic to a disk
  - Continuous bijection, distance does not matter
- **2-Manifold with boundary**: to a [half-]disk
- **3-Manifold, n-manifold, ...**
- **No singularities:**

![Diagram of 2-Manifolds](image)
Object topology

- Any manifold's topology is defined by a small set of numbers:
  - Surface: nb c of connected components + nb g of holes + nb b of boundaries
  - Volume: nb of conn. comp. + nb of tunnels + nb of cavities (bubbles) + nb of boundaries

- Euler formula for surface meshes:
  - \( V-E+F = \chi = 2(c-g)-b \)
  - \( \chi = \) Euler characteristic
  - g = genus
(Easy) Exercise

- Find the Euler characteristic of the following 6 surfaces:

- And for volumes?
Mesh data structures

- Ref.: *chapter 3* of Botsch et al.'s book
- How to store geometry and connectivity?
  - STL-like: store triangles, vertices duplicated
    => no connectivity
  - Shared vertex data structure (OBJ, OFF file formats): vertex list, triangles = triples of indices
    => no neighborhood info
  - Half-edge and variants
    => all is based on oriented edges
Half-edge data structure

- Three main classes:
  - **Vertex**
    - Coord, [id,] pointer to one outgoing half-edge
  - **Half-edge**
    - Pointers to the **origin** vertex, to the **next** and to the **opposite** half-edge, to the incident face
  - **Face**
    - Pointer to one incident half-edge

You can add whatever attributes you want (normal, color, ...)

Example: browsing the 1-ring neighborhood of a vertex

(1) start at a vertex
(2) find outgoing halfedge
(3) switch to opposite halfedge
(4) next halfedge points to neighbouring vertex
Example: browsing the 1-ring neighborhood of a vertex

Exercise:

- Write your own half-edge data structure:
  - class Vertex
  - class Edge
  - class Face

- Write a procedure `browseOneRing(Vertex* v)` which returns the 1-ring neighborhood of v as a list.
C++ libraries

- **CGAL** [http://www.cgal.org/](http://www.cgal.org/)
  - Developed by a consortium led by INRIA, lots of stuff
  - Widely used by researchers, tutorials
  - Somehow complicated (genericity)

- **OpenMesh** [http://www.openmesh.org/](http://www.openmesh.org/)
  - Developed by Mario Botsch at RWTH Aachen
  - Simpler, clearer
  - Lack of documentation

  - Why not?
Mesh processing

• Lots of work
  – Simplification
  – Smoothing, fairing
  – Parameterization
  – Remeshing
  – Deformation

• See Botsch et al.'s book

• Topic of the sessions 4 and 5 of this course
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Voxels

- Volumetric representation
- (Regularly) discretize the 3D space and only keep elements inside the object
- 2D: pixel = PICTure EElement
- 3D: voxel = VOlume EElement
- And also: surfel (surface), texel (texture), ...
Voxel set acquisition

- Using a function sampled on a grid
  - Numerical simulation
- Tomographic reconstruction (CT scan)
  - Medical area
- Depending on the acquisition/application, voxels contain scalar values (function, density, color, ...)
Octree

- Voxel hierarchy
- Saves memory
- Interesting for:
  - Spatial queries
  - Collision detection
  - Hidden surface removal ("view frustrum culling")

Courtesy S. Lefebvre
An introduction to discrete geometry

● Theoretical/Mathematical study of regular 2D/3D (simple) objects
  – Sampled on a grid
  – Object = point, line, plane

● How to define what is a line of voxels?

● Adapted algorithms
Why a regular grid

- Simple topology
- Easy address to a cell: coordinates
- Easy access from a cell to its neighbors
- Physical reality (sensors)
Cell

- Usually a convex polygon/polyhedron
- Regular
- The 3 principal cases: square/cube, hexagon/hexahedron, triangle/tetrahedron

Courtesy D. Coeurjolly & I. Sivignon
Advantage of squares/cubes

- **Square:**
  - 4 neighbors
  - 1 configuration
- **Triangle:**
  - 3 neighbors
  - 2 configurations
- **Hexagon:**
  - 6 neighbors
  - 2 configurations

Courtesy D. Coeurjolly & I. Sivignon
Adjacency on a voxel grid

• (Combinatorial) Def.:
  - 6-neighbors = voxels that share a face
  - 18-neighbors = voxels that share a edge
  - 26-neighbors = voxels that share a vertex

Courtesy D. Coeurjolly & I. Sivignon
Adjacency on a voxel grid

• (Topological) Def.:
  - 2-neighbors = voxels that share a face
  - 1-neighbors = voxels that share a edge
  - 0-neighbors = voxels that share a vertex

Courtesy D. Coeurjolly & I. Sivignon
Basic discrete geometry definitions

- An ordered set \( \{c_1, ..., c_n\} \) of discrete cells is a (topological) \( k \)-path if \( \forall i, c_i \) is a \( k \)-neighbor of \( c_{i-1} \)
- It is a \( k \)-arc if \( \forall i, c_i \) has exactly two \( k \)-neighbors
- It is a \( k \)-curve if it a \( k \)-arc + \( c_1 = c_n \)
- A set \( O \) of discrete cells is a \( k \)-object if \( \forall c, c' \) in \( O \), one can find a \( k \)-path from \( c \) to \( c' \) in \( O \)
Discrete object boundary

- Problem with discrete objects: their boundary is not obvious

Inside or outside? One or two components?
Problem

- **Jordan's theorem**: every smooth (n-1)-manifold in $\mathbb{R}^n$ disjoins space into two connected domains (the **inside** and the **outside**); it is the common **boundary** of these domains.

- **Corollary**: impossible to find a path from inside to outside.

- Need to define the right adjacency!
Adjacency couple

- Need to define one connexity for the (inside) object, and one for the outside

- **Exercise:** possible couples?
Adjacency couple

- Need to define one connexity for the (inside) object, and one for the outside

- Possible couples: (6, 18), (6, 26), (18, 6) and (26, 6)
Contour

- **Def.:** connected set of cell *faces* between a cell inside the object and a cell outside

- Coherent with Jordan; depends on the chosen adjacency

- Contour of a volume = surface (to display)
Contour coding

• We want the code to be:
  – **Compact**: compared to a simple list of the discrete faces coordinates
  – **Toggle**: the surface can be reconstructed from the code
  – **Invariant**: w.r.t. some geometrical transforms
  – **Informative**: about the surface (area, …)

• In 2D: **Freeman code**
Freeman code

- **Idea:** code the path between two consecutive pixels of the discrete curve
Properties

- Reversible (unicity)
- Geometrical transforms does not affect much the code
  - Translation: just change the origin point
  - Rotation with angle $\pi/2$: $c' = c + 2 \mod 8$ (if 8-adjacency)
- Can give an estimate of the curve length
  - $L := L + 1$ if $c$ is even
  - $L := L + \sqrt{2}$ if $c$ is odd
Discrete line (2D)

• How to define a discrete line from a real line?

• Bresenham algorithm:
  – Choose the closest pixel to the line in the vertical direction (incremental)
Other definition

- Let $D: y = ax + b$ be the real line. $D$ is the set of points/pixels $p_i = (x_i, y_i)$ with $x_i = i$ and $y_i = \lfloor ax_i + b + 0.5 \rfloor$.

- **Properties:**
  - $D$ is a 8-arc
  - $D$ can be Freeman-coded with codes 0 and 1 only
  - If $a$ is rational, then the code of $D$ is periodic
Euclidean 1: given two points A and B, there exists only one line going through A and B.
Discrete vs. continuous

- **Euclide 2**: Two non parallel lines intersect exactly once.
Third definition [Reveillès 1991]

- **Arithmetic discrete line:**
  - \( D(a,b,d,e) = \{ (x, y) \text{ with } x, y, a, b, d, e \in \mathbb{Z}, \ b \neq 0, \ 0 \leq ax - by + d < e \text{ and } \gcd(a, b) = 1 \} \).
  - \( a/b \) is the line slope, \( d \) is the origin offset and \( e \) the thickness.

- **Exercise:**
  - Draw a regular grid.
  - Draw (the beginning of) the following lines: \( D(3,7,0,5), D(3,7,0,7), D(3,7,0,8), D(3,7,0,10) \) and \( D(3,7,0,16) \).
Third definition [Reveillès 1991]

- **Arithmetic discrete line:**
  
  - \( D(a,b,d,e) = \{ (x, y) \text{ with } x,y,a,b,d,e \text{ in } \mathbb{Z}, b \neq 0, 0 \leq ax - by + d < e \text{ and } \gcd(a,b)=1 \} \).
  
  - \( a/b \) is the line slope, \( d \) is the origin offset and \( e \) the thickness.

Courtesy D. Coeurjolly and I. Sivignon
Properties

Let $D(a,b,d,e)$ be a discrete line. Then:

- if $e < \max(|a|,|b|)$ then $D$ is disconnected;
- if $e = \max(|a|,|b|)$ then $D$ is a 8-arc and is called a naive line;
- if $\max(|a|,|b|) < e < |a|+|b|$ then $D$ has both 4- and 8-connected parts;
- if $e = |a|+|b|$ then $D$ is a 4-arc and is called a standard line;
- else $D$ is called a thick line.
Properties

Let $D$ be the real line $ax-by+d = 0$ with $a, b, d$ in $\mathbb{Z}$; suppose $|a| \leq |b|$. Then:

- the **default discretization** of $D$, that is to say the set $\{ (x, y), y = \lfloor (-ax-d)/b \rfloor \}$ is exactly $D(a, b, d, b)$;

- the **excess discretization** of $D$, that is to say the set $\{ (x, y), y = \lceil (-ax-d)/b \rceil \}$ is exactly $D(a, b, d+b-1, b)$;

- ...
Discrete plane (3D)

- **Discretization of a real plane:**
  - Let \( d: z = ax+by+c \) be the real plane. \( P \) is the set of points/voxels \( p = (x,y,z) \) with \( x \) and \( y \) in \( \mathbb{Z} \) and \( z = \lfloor ax + by + c \rfloor \).

- **Arithmetic discrete plane:**
  - \( P(a,b,c,d,e) = \{ (x, y, z) \text{ with } x,y,z,a,b,c,d,e \text{ in } \mathbb{Z}, d \leq ax + by + cz < d + e \text{ and } \gcd(a,b,c)=1 \} \).

  - \( (a, b, c)^\top \) is the plane normal, \( d \) is the origin offset and \( e \) the thickness.
Some discrete planes

P(6,13,27,0,15)  P(6,13,17,0,27)  P(6,13,17,0,46)

Courtesy D. Coeurjolly and I. Sivignon
Discrete geometry

This part was inspired by a course given by David Coeurjolly and Isabelle Sivignon (CNRS researchers, LIRIS, Lyon)
Books

  - Available at INRIA or University library
The end

- Next week:
  - Parametric curves and surfaces
  - Subdivision surfaces
  - Lecturer: Marie-Paule Cani

- These slides will be available on the course's webpage:
  http://evasion.imag.fr/Membres/Franck.Hetroy/Teaching/Geo3D/